Conic Enrichment

1.) $e = \frac{c}{a}$

$ae = c$

$a^2 - b^2 = c^2$

$a^2 - b^2 = a^2 e^2$

$-b^2 = a^2 e^2 - a^2$

$-b^2 = a^2(e^2 - 1)$

$b^2 = a^2(1 - e^2)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{a^2(1-e^2)}$$

2) $pt \ (c, y)$

$$\frac{c^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{a^2 - b^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = \frac{a^2}{a^2} - \frac{a^2 - b^2}{a^2}$$

$$\frac{y^2}{b^2} = \frac{b^2}{a^2}$$

$$y^2 = \frac{b^4}{a^2}$$

$$y = \frac{b^2}{a}$$

$$LR = 2y = \frac{2b^2}{a}$$

3) $2a = 10$

$a = 5$

$b < 5$

the ellipse becomes more circular

4) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\frac{x^2}{a^2} + \frac{x^2}{b^2} = 1$$

$$\frac{x^2 b^2 + a^2 x^2}{a^2 b^2} = 1$$

$$\frac{x^2(b^2 + a^2)}{a^2 b^2} = 1$$

$$x^2 = \frac{a^2 b^2}{(b^2 + a^2)}$$

$$A = (2x)^2 = 4x^2$$

$$= \frac{4a^2 b^2}{b^2 + a^2}$$

5) $d_1 + d_2 \leq 20$

a) so the definition of an ellipse describes the outer bound the boat can travel The islands are the foci

b) Island 1 $(-6, 0)$
Island 2 $(6, 0)$

c) 20 miles $(-10, 0)$ $(10, 0)$

d) $10^2 - b^2 = 6^2$
   $b^2 = 64 \quad b = 8$ $\frac{x^2}{100} + \frac{y^2}{64} = 1$

6) $m(6, 2)$

$2a = 6$

$a = 3 \quad c = 4$

$$\frac{(x-6)^2}{9} - \frac{(y-2)^2}{7} = 1$$