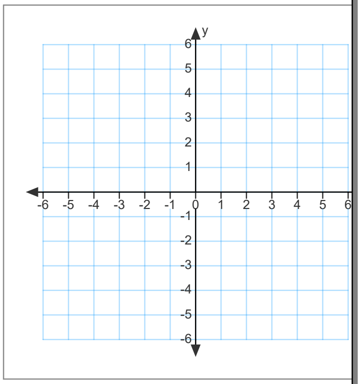
Warm up

1. Find the rectangular equation with the following parameters. Make a sketch and state the domain and range of the equation.



a. $x = t$

$y = t + 3$

$Y = x + 3$

b. $x = \sqrt{t+1}$

$y = t - 2$

t bucket

$[-1, \infty)$

$D_x\ [-1, \infty)$

$R_x\ [0, \infty) \Rightarrow D_y$

D_y ~~$(-\infty, \infty)$~~ $[-1, \infty)$

R_y ~~$(-\infty, \infty)$~~ $[-3, \infty)$

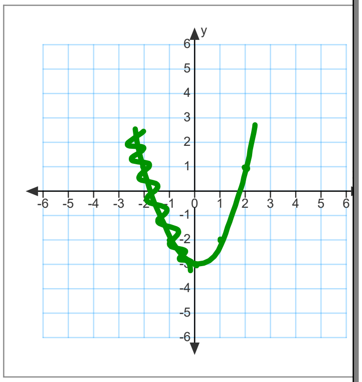
$\downarrow$

R_y

$x^2 - 1 = t$

$Y = x^2 - 1 - 2$

$Y = x^2 - 3$



Apr 23-8:59 AM

GO COUGARS!

p 776 Homework Questions

In Exercises 3–22, (a) sketch the curve represented by the parametric equations (indicate the orientation of the curve) and (b) eliminate the parameter and write the corresponding rectangular equation whose graph represents the curve. Adjust the domain of the resulting rectangular equation if necessary.

3. $x = 3t - 3$
   $y = 2t + 1$
5. $x = \frac{1}{4}t$
   $y = t^2$
7. $x = t + 2$
   $y = t^2$
9. $x = t + 1$
   $y = \frac{t}{t+1}$
11. $x = 2(t + 1)$
   $y = |t - 2|$
13. $x = 3\cos\theta$
   $y = 3\sin\theta$
19. $x = e^{-t}$
   $y = e^{3t}$
21. $x = t^3$
   $y = 3\ln t$

$x - 1 = t$

$Y = \frac{x-1}{x}$

$Y = 1 - \frac{1}{x}$

$= -\frac{1}{x} + 1$

D_x

$D_y\ (-\infty, -1)(-1, \infty)$

$D_x\ (-\infty, \infty)$

$R_x\ (0, \infty)$

$x = e^{-t}$

$\ln x = -t$

$-\ln x = t$

$Y = e^{-3\ln x}$

$Y = e^{\ln x^{-3}}$

$Y = x^{-3}$

In Exercises 23 and 24, determine how the plane curves differ from each other.

23. (a) $x = t$
   $y = 2t + 1$
   (c) $x = e^{-t}$
   $y = 2e^{-t} + 1$

In Exercises 37–44, find a set of parametric equations for the rectangular equation using (a) $t = x$ and (b) $t = 2 - x$.

37. $y = 3x - 2$
39. $y = x^2$
41. $y = x^2 + 1$
43. $y = \frac{1}{x}$

Feb 2-9:51 PM

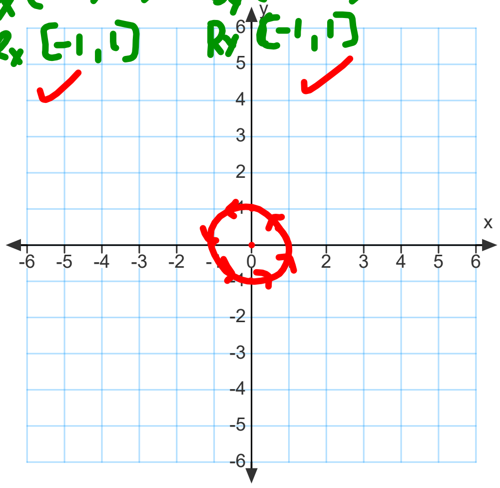
10.6 Parametric Equations Day 2

Apr 19-9:11 AM

x=cost y=sint

$D_x\ (-\infty,\infty)$ $D_y\ (-\infty,\infty)$

$R_x\ [-1,1]$ ✓ $R_y\ [-1,1]$ ✓



t	x	y
0	1	0
$\frac{\pi}{2}$	0	1
π	-1	0
$\frac{3\pi}{2}$	0	-1
2π	1	0

D[-1,1]
R[-1,1]

$$x^2 = \cos^2 t$$
$$+\ y^2 = \sin^2 t$$
$$x^2 + y^2 = \cos^2 t + \sin^2 t$$
$$x^2 + y^2 = 1$$

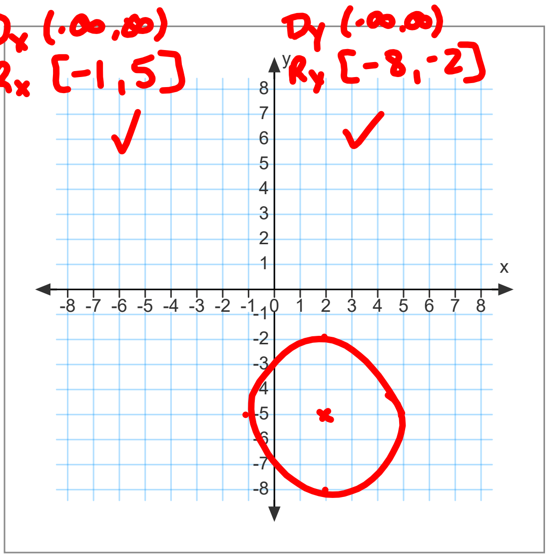
circle
center (0,0)
radius 1

Apr 19-9:13 AM

x = 2+3cost   y = -5+3sint

D_x $(-\infty,\infty)$ D_y $(-\infty,\infty)$

R_x $[-1,5]$ ✓ R_y $[-8,-2]$ ✓



t	x	y
0	5	-5
$\frac{\pi}{2}$	2	-2
π	-1	-5
$\frac{3\pi}{2}$	2	-8
2π	5	-5

D $[-1,5]$
R $[-8,-2]$

$\frac{x-2}{3} = \cos t$

$\frac{y+5}{3} = \sin t$

$\frac{(x-2)^2}{9} + \frac{(y+5)^2}{9} = 1$

$(x-2)^2 + (y+5)^2 = 9$

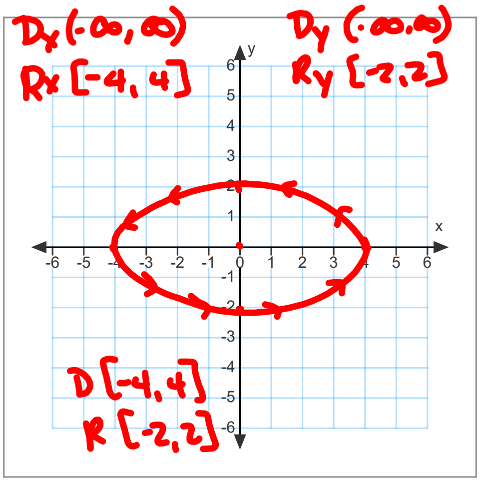
Circle C $(2,-5)$
radius = 3

Apr 19-9:16 AM

x = 4cost   y = 2sint

D_x $(-\infty,\infty)$ D_y $(-\infty,\infty)$

R_x $[-4,4]$ R_y $[-2,2]$



D $[-4,4]$
R $[-2,2]$

t	x	y

$\frac{x}{4} = \cos t$

$\frac{y}{2} = \sin t$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

m = $(0,0)$
a = 4
b = 2

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Get in pairs and work the following problems together.

In Exercises 25–28, eliminate the parameter and obtain the standard form of the rectangular equation.

25. Line through (x_1, y_1) and (x_2, y_2):

$x = x_1 + t(x_2 - x_1),\ y = y_1 + t(y_2 - y_1)$

26. Circle: $x = h + r\cos\theta,\ y = k + r\sin\theta$

27. Ellipse: $x = h + a\cos\theta,\ y = k + b\sin\theta$

28. Hyperbola: $x = h + a\sec\theta,\ y = k + b\tan\theta$

$\sec^2\theta - \tan^2\theta = 1$

$\frac{x-h}{a} = \sec\theta \quad \frac{y-k}{b} = \tan\theta$

28 $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

25 $\frac{x-x_1}{x_2-x_1} = t$

$y = y_1 - \left(\frac{x-x_1}{x_2-x_1}\right)(y_2 - y_1)$

Apr 21-12:53 PM

Work Backwards - find a set of parametric equations that represent the line or conic.

for lines use

$x = x_1 + t(x_2 - x_1) \qquad y = y_1 + t(y_2 - y_1)$

for circles and ellipeses use

$x = a + r\cos t \qquad y = a + r\sin t$

'r' represents distance from center

'a' represents center

if r's are same the conic is a circle

if r's are different the conic is an ellipse

trig functions that represent a hyperbola

are sec and tan

but order here matters:

Parametric Equation of a Hyperbola

VERTICAL	HORIZONTAL
$x = h + b\tan t$ $y = k + a\sec t$	$x = h + a\sec t$ $y = k + b\tan t$

center **(h, k)**
length of transverse axis- **2a**
length of conjugate axis- **2b**

Examples: Find a pair of parametric equations for the given line or conic.

1 2

line: line passes through (0, 0) and (6, -3)

$x = 0 + t(6) \rightarrow x = 6t$ $y = 0 + t(-3) \rightarrow y = -3t$

circle: center (-3, 2), radius = 5

$x = -3 + 5\cos t$ $y = 2 + 5\sin t$

ellipse: vertices (4, 7) and (4,-3)

foci (4, 5) and (4, -1)

m(4,2) vertical

$a = 5$
$b = 4$
$c = 3$

$x = 4 + 4\cos t$
$y = 2 + 5\sin t$

hyperbola: vertices (± 2, 0), foci (± 4, 0)

Apr 22-10:35 AM

HOMEWORK

p 776 13-17 odd, 29-35 odd,

Workbook p 113 1-21 odds

Feb 2-9:51 PM

May 2-1:35 PM