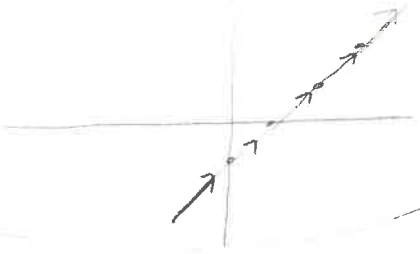
Parametric Equations Worksheet Key

1) $x = 1 + t$, $y = t$ $\Rightarrow x = 1 + y \Rightarrow y = x - 1$

$D_x: (-\infty, \infty)$, $D_y: (-\infty, \infty)$ $\}$ $(-\infty, \infty)$

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

t	x	y
-2	-1	-2
-1	0	-1
0	1	0
1	2	1
2	3	2



2) $x = 2t - 3$ $\Rightarrow \frac{x+3}{2} = t$

$y = 9 - 4t$

$y = 9 - 4\left(\frac{x+3}{2}\right)$

$= 9 - 2x - 6$

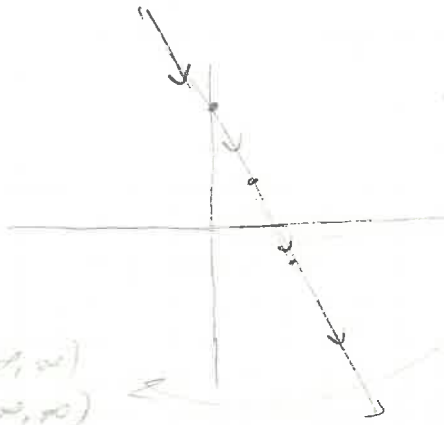
$= -2x + 3$

$D_x: (-\infty, \infty)$

$D_y: (-\infty, \infty)$

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$

t	x	y
-2	-7	17
-1	-5	13
0	-3	9
1	-1	5
2	1	1



3) $x = 4\sin t$ $\quad y = 4\cos t$

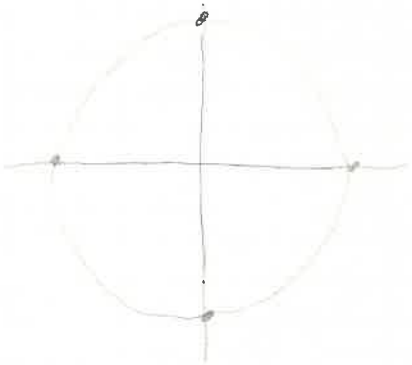
$x^2 = 16\sin^2 t$ $\quad y^2 = 16\cos^2 t$

$16\sin^2 t + 16\cos^2 t = 16$

$x^2 + y^2 = 16$ $\quad$ circle $C(0,0)$ $r = 4$

D: $[-4, 4]$
R: $[-4, 4]$

t	x	y
$-\frac{\pi}{2}$	-4	0
0	0	4
$\frac{\pi}{2}$	4	0
π	0	-4



4) $x = t + 2$ $\quad y = \frac{2}{t}$

$x - 2 = t$ $\quad y = \frac{2}{x-2}$

D: $(-\infty, \infty)$

$D_y: (-\infty, 0) \cup (0, \infty)$

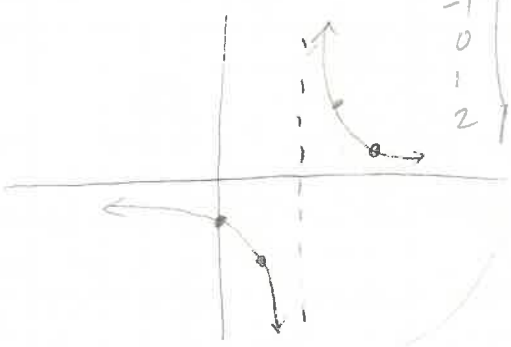
VA: $x = 2$

$(-\infty, 0)$ HA: $y = 0$

$(0, \infty)$ y-int: -1

D: $(-\infty, 2) \cup (2, \infty)$
R: $(-\infty, 0) \cup (0, \infty)$

t	x	y
-2	0	-1
-1	1	-2
0	2	und
1	3	2
2	4	1



5) $x = 4t - 2$ $\quad y = 8t^2$

$\frac{x+2}{4} = t \Rightarrow y = 8\left(\frac{x+2}{4}\right)^2$

$= 8\left(\frac{(x+2)^2}{16}\right)$

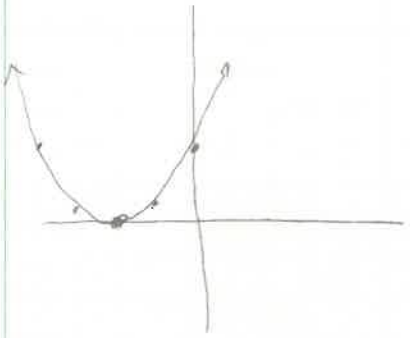
$= \frac{1}{2}(x+2)^2$

$D_x: (-\infty, \infty)$

$D_y: (-\infty, \infty)$

$R_x = (-\infty, \infty)$

$R_y = [0, \infty)$



t	x	y
-2	-10	18
-1	-6	8
0	-2	0
1	2	8
2	6	32

D: $(-\infty, \infty)$
R: $[0, \infty)$

6) $x = t - 3$ $\quad y = \sqrt{t-2}$

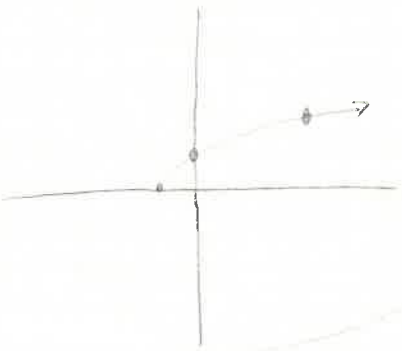
$x + 3 = t$ $\quad y = \sqrt{x+1}$

$D_x: (-\infty, \infty)$ $\quad [3, \infty)$

$D_y: [-1, \infty)$

R_x $[-1, \infty)$

R_y $[0, \infty)$



t	x	y
2	-1	0
3	0	1
6	3	2

D: $[-1, \infty)$
R: $[0, \infty)$

7) $x = 4\cos t$ $\quad y = 8\sin t$

$x^2 = 16\cos^2 t$ $\quad y^2 = 64\sin^2 t$

$\frac{x^2}{16} = \cos^2 t$ $\quad \frac{y^2}{64} = \sin^2 t$

$\frac{x^2}{16} + \frac{y^2}{64} = 1$ $\quad$ Ellipse

$a = 8$

$b = 4$

vertical

t	x	y
$-\frac{\pi}{2}$	0	-8
0	4	0
$\frac{\pi}{2}$	0	8
π	-4	0

D: $[-4, 4]$

R: $[-8, 8]$

8) $x = 5 - 3t$

$\frac{x-5}{-3} = t$

$y = 2 + t$

$= 2 + \frac{-x+5}{3}$

$= \frac{6 - x + 5}{3}$

$= \frac{-x+11}{3}$

$= -\frac{1}{3}x + \frac{11}{3}$

Dx $(-\infty, \infty)$ } $(-\infty, \infty)$

Dy $(-\infty, \infty)$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

t	x	y
-2	11	0
-1	8	1
0	5	2
1	2	3
2	-1	4

9) $x = 3t - 1$

$\frac{x+1}{3} = t$

$y = t^2 + 2$

$= \left(\frac{x+1}{3}\right)^2 + 2$

$= \frac{1}{9}(x+1)^2 + 2$

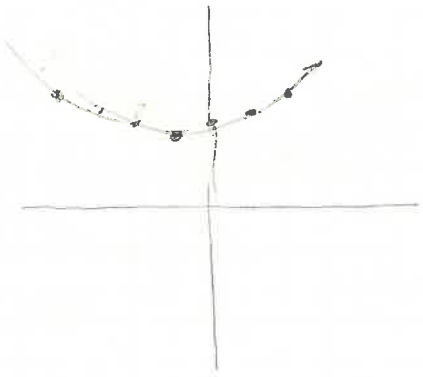
$= V(-1, 2)$

Dx $(-\infty, \infty)$ $(-\infty, \infty)$

Dy $(-\infty, \infty)$

D: $(-\infty, \infty)$

R: $[2, \infty)$



t	x	y
-2	-7	6
-1	-4	3
0	-1	2
1	2	3
2	5	6

10) $x = |t|$ $\quad y = t$ $\quad x = |y|$

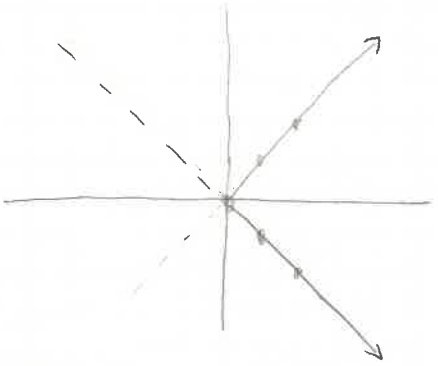
$x = \pm t$ $\quad y = \pm x$

$D_x : (-\infty, \infty)$ $\quad D_y : (-\infty, \infty)$

$R_x : [0, \infty)$ $\quad R_y = (-\infty, \infty)$

t $(-\infty, \infty)$

t	x	y
-2	2	-2
-1	1	-1
0	0	0
1	1	1
2	2	2



D: $[0, \infty)$

R: $(-\infty, \infty)$

11) $x = \sqrt{t}$ $\quad y = \sqrt{t}$ $\quad x^2 = t$

$y = \sqrt{x^2}$

$y = \pm x$

D $(0, \infty)$

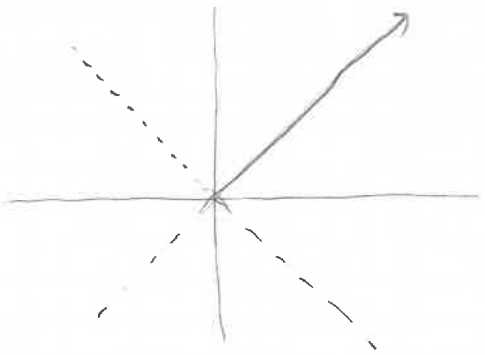
R $[0, \infty)$

D: $[0, \infty)$

R: $[0, \infty)$

t	x	y
0	0	0
1	1	1
4	2	2
9	3	3

t $[0, \infty)$



D $[0, \infty)$

R $[0, \infty)$

12) $x = \sqrt{t}$ $\quad y = 1 - t$ $\quad x^2 = t$

$y = 1 - x^2$

D: $[0, \infty)$

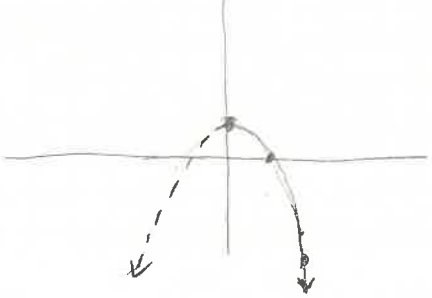
R: $[0, \infty)$

D $(-\infty, \infty)$

R: $(-\infty, 1]$

t $[0, \infty)$

t	x	y
0	0	1
1	1	0
4	2	-3
9	3	-8



D: $[0, \infty)$

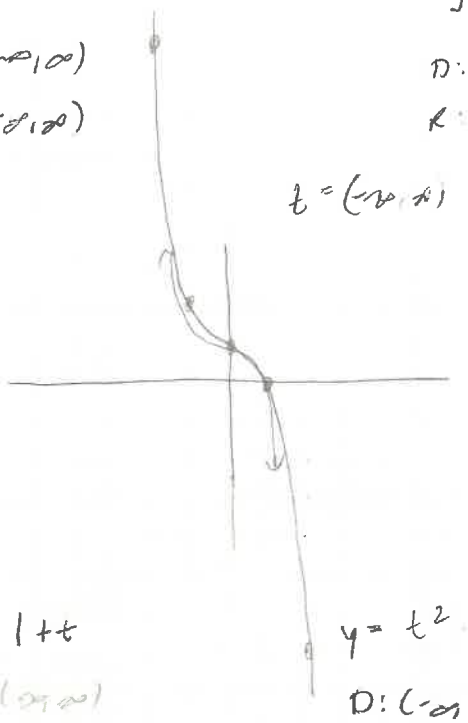
R: $[-\infty, 1]$

13) $x = \sqrt[3]{t}$ $\quad y = 1 - t$ $\quad x^3 = t$

D: $(-\infty, \infty)$ $\quad$ D: $(-\infty, \infty)$ $\quad y = 1 - x^3$

R: $(-\infty, \infty)$ $\quad$ R: $(-\infty, \infty)$ $\quad y = -x^3 + 1$

$t = (-\infty, \infty)$



D: $(-\infty, \infty)$

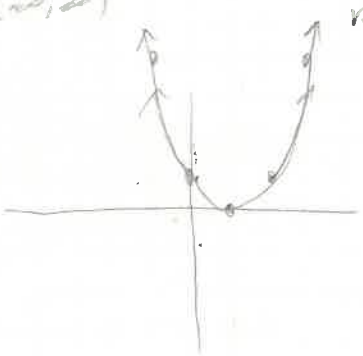
R: $(-\infty, \infty)$

t	x	y
-8	-2	9
-1	-1	2
0	0	1
1	1	0
8	2	-7

14) $x = 1 + t$ $\quad y = t^2$ $\quad t = x - 1$

D: $(-\infty, \infty)$ $\quad$ D: $(-\infty, \infty)$ $\quad y = (x-1)^2$

R: $(-\infty, \infty)$ $\quad$ R: $[0, \infty)$ $\quad t (-\infty, \infty)$



D: $(-\infty, \infty)$

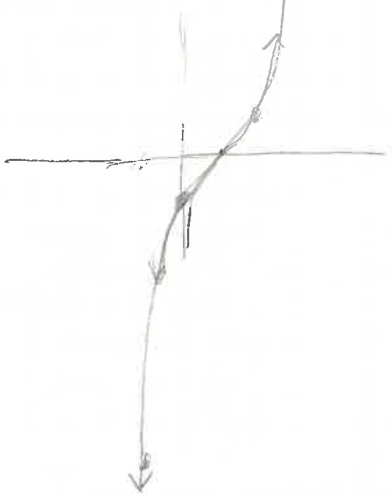
R: $[0, \infty)$

t	x	y
-2	-1	4
-1	0	1
0	1	0
1	2	1
2	3	4

15) $x = t + 1$ $\quad y = t^3$ $\quad x + 1 = t$ $\quad y = (x-1)^3$

D: $(-\infty, \infty)$ $\quad$ D: $(-\infty, \infty)$ $\quad t (-\infty, \infty)$

R: $(-\infty, \infty)$ $\quad$ R: $(-\infty, \infty)$



D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

t	x	y
-2	-1	-8
-1	0	-1
0	1	0
1	2	1
2	3	8

16) $x = \sqrt{1-t}$ $y = \sqrt{t}$

D: $(-\infty, 1]$ D: $(0, \infty)$ $[0,1]$

R: $[0, 1]$ R: $[0, 1]$

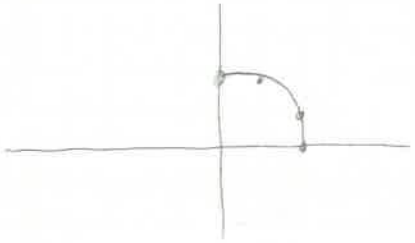
$x^2 = 1 - t$ $y^2 = t$

$x^2 - 1 = -t$

$-x^2 + 1 = y^2$

$1 = x^2 + y^2$

t	x	y
0	1	0
3/4	1/2	√3/2
1	0	1
1/4	√3/2	1/2



D: $[0, 1]$

R: $[0, 1]$

17) $x = \sqrt{1-t^2}$

$x = \sqrt{-t^2+1}$

D $[-1, 1]$

R $[0, 1]$

$y = t$

D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

$t [-1, 1]$

$x^2 = 1 - t^2$

$x^2 - 1 = -t^2$

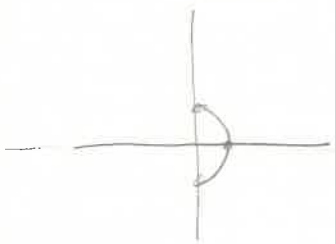
$1 - x^2 = t^2$

$1 - x^2 = y^2$

$x^2 + y^2 = 1$

$y^2 = t^2$

t	x	y
-1	0	1
0	1	0
1	0	1



D: $[0, 1]$

R: $[-1, 1]$

18) $x = \cos t$

D: $(-\infty, \infty)$

R: $[-1, 1]$

$y = 3 \sin t$

D: $(-\infty, \infty)$

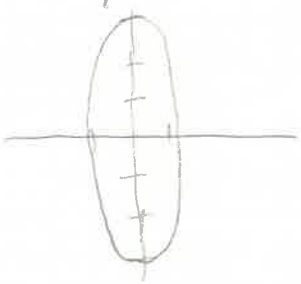
R: $[-3, 3]$

$\frac{x^2}{1} + \frac{y^2}{9} = 1$

t	x	y
$-\frac{\pi}{2}$	0	-3
0	1	0
$\frac{\pi}{2}$	0	3
π	-1	0

$x^2 = \cos^2 t$

$\frac{y^2}{9} = \sin^2 t$



D: $[-1, 1]$

R: $[-3, 3]$

19) $x = -\sqrt{1-t}$ $\quad y = -\sqrt{t}$

$= -\sqrt{-(t+1)}$

b: $(-\infty, 1]$ $\quad D: (0, \infty) \Rightarrow [0,1)$

R: $[-1, 0]$, $\quad R: [-1, 0]$

t	x	y
0	-1	0
1/2	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
3/4	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$
1	0	-1

$x^2 = 1 - t$ $\quad y^2 = t$

$x^2 - 1 = -t$

$-x^2 - 1 = t$ $\quad y^2 = -x^2 - 1$

$x^2 + y^2 = 1$

D: $[-1, 0]$

R: $[-1, 0]$

20) $x = 2 + \sin t$ $\quad y = 3 + \cos t$

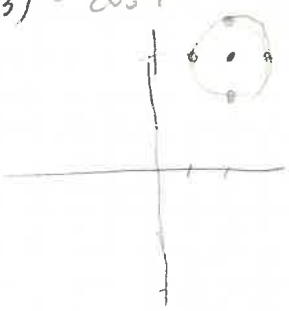
D: $(-\infty, \infty)$ $\quad D: (-\infty, \infty)$ $\quad t\ (\infty, \infty)$

R: $[1, 3]$ $\quad R: [2, 4]$

t	x	y
$-\pi/2$	1	3
0	2	4
$\pi/2$	3	3
π	2	2

$(x-2)^2 = \sin^2 t$ $\quad (y-3)^2 = \cos^2 t$

$(x-2)^2 + (y-3)^2 = 1$



D: $[1, 3]$

R: $[2, 4]$

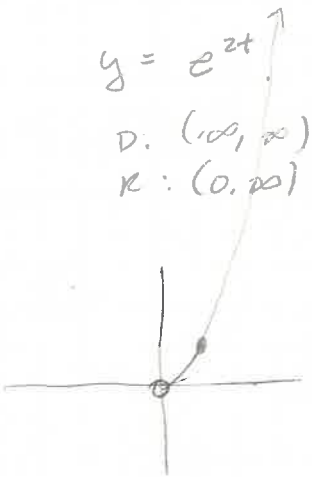
21) $x = e^t$ $\quad y = e^{2t}$

D: $(-\infty, \infty)$ $\quad D: (-\infty, \infty)$ $\quad t\ (-\infty, \infty)$

R: $(0, \infty)$ $\quad R: (0, \infty)$ $\quad y = x^2$

$x^2 = e^{2t}$

t	x	y
-2	.135	.02
-1	.368	.14
0	1	1
1	2.7	7.4
2	7.4	54.6



D: $(0, \infty)$

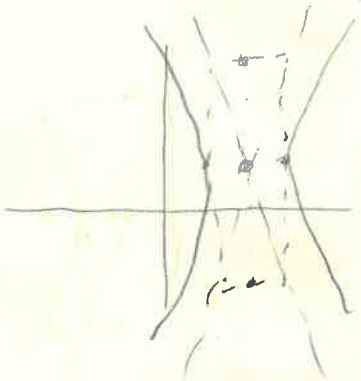
R: $(0, \infty)$

$D(-\infty,\infty)\ R(-\infty,1]\cup[3,\infty)$ | $(-\infty,\infty)R(-\infty,\infty)$

22. $x = 2+\sec t \qquad y = 1+3\tan t$

$\frac{(x-2)^2}{1} = \sec^2 t \qquad \frac{(y-1)^2}{9} = \tan^2 t$

$\frac{(x-2)^2}{1} - \frac{(y-1)^2}{9} = 1$



$D\ (-\infty,1]\cup[3,\infty)$

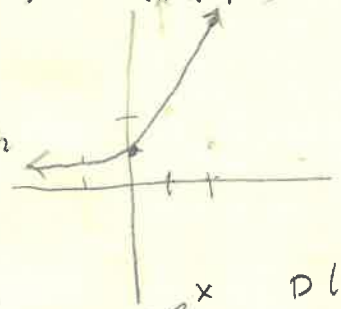
$R\ (-\infty,\infty)$

23. $x = 2\ln t \qquad y = t^2$

$D\ (0,\infty)$ $D\ (0,\infty)$

$R\ (-\infty,\infty)$ $R\ (0,\infty)$

1	0	1
2.7	2	7.3



$x = \ln t^2$

$e^x = t^2$

$y = e^x \qquad D\ (-\infty,\infty)$

$R\ (0,\infty)$

24. $x = \sin^2 t \qquad y = \cos^2 t$

$D\ (-\infty,\infty)$ $D\ (-\infty,\infty)$

$R\ [0,1]$ $R\ [0,1]$

t	x	y
0	0	1
$\frac{\pi}{2}$	1	0
π	R	1
$\frac{3\pi}{2}$	1	0
2π	0	1

$x+y=1$

$y = -x+1 \qquad D\ [0,1]$

$R\ [0,1]$