Practice Exercises: Identities and graphs

In Exercises 1–4, write each expression in terms of sin θ and cos θ and simplify.

1) $\csc\theta \cot\theta$

2) $\sec\theta \tan\theta \csc\theta$

3) $\dfrac{\csc\theta}{\sec\theta}$

4) $\dfrac{\sec\theta}{\tan\theta}$

Add or subtract as indicated. Then simplify your answers if possible.

5) $\dfrac{\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{\cos\theta}$

6) $\dfrac{1}{\cos\theta} - \cos\theta$

In Exercises 7–10, show that the equation is an identity by showing that the left side and the right side are the same, or state a known identity.

7) $(\sin\theta - \cos\theta)^2 = 1 - 2\sin\theta\cos\theta$

8) $\dfrac{\sin\theta}{\csc\theta} = \sin 2\theta$

9) $\dfrac{\csc\theta \tan\theta}{\sec\theta} = 1$

10) $\sec\theta(\sin\theta + \cos\theta) = \tan\theta + 1$

8.1 Exercises: Show that each of the following equations are identities.

1) $\cos x \tan x = \sin x$

2) $\tan x \cot x = 1$

3) $\tan x \cos x \csc x = 1$

4) $\sin x \cos x \tan x = 1 - \cos^2 x$

5) $\cos^2 x - \sin^2 x = 1 - 2\sin^2 x$

6) $\cos x (\sec x - \cos x) = \sin^2 x$

7) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

8) $(\sin x - \cos x)^2 = 1 - 2\sin x \cos x$

9) $\sec^2 x - \tan^2 x = 1$

10) $\sec^2 x + \csc^2 x = \sec^2 x \csc^2 x$

11) $\dfrac{1 + \cos x}{\sin x} = \dfrac{\sin x}{1 - \cos x}$

12) $\dfrac{1}{\sin x \cos x} - \tan x = \cot x$

13) $\dfrac{1}{1 - \sin x} + \dfrac{1}{1 + \sin x} = 2\sec^2 x$

14) $\sec x \csc x - \cot x = \tan x$

15) $\dfrac{\sin x}{1 - \cos x} + \dfrac{1 - \cos x}{\sin x} = \dfrac{2}{\sin x}$

16) $\tan x + \cot x = \sec x \csc x$

17) $\dfrac{\sec x}{\cos x} + \dfrac{\csc x}{\sin x} = \sec^2 x \csc^2 x$

18) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

19) $\dfrac{2\sin x \cos x}{\cos^2 x - \sin^2 x} = \dfrac{2\tan x}{1 - \tan^2 x}$

1) Without using a calculator, find all angles x such that $0 \le x < 2\pi$ that solve the equation. (You will often be asked to find the exact value or not be permitted a calculator. In this case, the angles involved are special angles based on the 30°-60°-90°, the 45°-45°-90° triangles, or the quadrantal angles whose sin or cos is 0, 1, or –1. Since these are the only trig equations that can be solved easily without a calculator, they occur with disproportionate frequency and you should be especially adept at solving them.

a) $\sin x = \frac{\sqrt{3}}{2}$

b) $\cos x = 1$

c) $\cos x = -\frac{1}{2}$

d) $\sin x = -1$

e) $\sin x = -\frac{\sqrt{2}}{2}$

f) $\cos x = 0$

g) $2\cos x = -\sqrt{3}$

h) $2\sin x = -\sqrt{2}$

i) $\sin x = 0$

j) $2 \sin x = 1$

k) $5 \sin x = 5$

l) $6 \cos x - 3 = 0$

2) Find all angles x such that $0 \le x < 2\pi$ that solve the following equations. Round answers to three decimal places.

a) $\cos x = -0.1724$

b) $\sin x = 0.5544$.

3) Find all angles that solve the following equations. Give exact answers if possible or round answers to three decimal places.

a) $\sin x = -0.3456$

b) $6 \cos x = 3$

c) $\cos x = 0.895534$

d) $2 \sin x + \sqrt{3} = 0$

e) $3 \sin x = -1$

f) $11 \cos x = 3$

g) $\sin x = -1$

h) $36 \cos x = 7$

i) $17 \sin x - 8 = 0$

j) $17 \cos x - 15 = 0$

7.2 Exercises: In Exercises 4–21, find all values of x such that $0 \le x < 2\pi$ that solve the equation. If possible, find exact answers. Otherwise round answers to 3 decimal places.

4) $4 \cos x + 1 = 0$

5) $5 \sin x = 8$ (Be careful!)

6) $\sin^2 x = 1$

7) $36 \cos x + 7 = 0$

8) $5 \cos x + 3 = 0$

9) $25 \sin x + 7 = 0$

10) $4 \cos^2 x - 1 = 0$

11) $2 \sin^2 x = 1$

12) $\sin 4x = -\frac{1}{2}$

13) $2 \cos^2 x - \cos x - 1 = 0$

14) $6 \sin^2 x - \sin x - 1 = 0$

15) $\cos 2x = \frac{1}{2}$

16) $2 \sin 3x = -1$

17) $\cos 4x = -1$

18) $4 \cos^2 (x - \frac{\pi}{4}) - 2 = 0$

19) $5 \sin 2(x - 1) + 3 = 0$

20) $2 \sin^2 x + \sin x = 0$

21) $2 \sin^2 3x + \sin 3x = 0$