### 5.1 Using Fundamental Identities

# Pythagorean Identities Simplifying Trigonometric Expressions Using Factoring and Foiling to Simplify 

Pythagorean Identities

$\frac{\sin ^{2} x}{\cos ^{2} x}+\frac{\cos ^{2} x}{\cos ^{2} x}=\frac{1}{\cos ^{2} x} \quad \frac{\sin ^{2} x}{\sin ^{2} x}+\frac{\cos ^{2} x}{\sin ^{2} x}=\frac{1}{\sin ^{2} x}$
$\sin ^{2} x+\cos ^{2} x=1$ $\tan ^{2} x+1=\sec ^{2} x$
$1+\cot ^{2} x=\csc ^{2} x$
$\sin ^{2} x=1-\cos ^{2} x$
$\sec ^{2} x=\tan ^{2} x+1$
$1-\csc ^{2} x=-\cot ^{2} x$
$\cos ^{2} x=1-\sin ^{2} x$
$\sec ^{2} x-\tan ^{2} x=1$
$1=\csc ^{2} x-\cot ^{2} x$ :
$\sin ^{2} x-1=-\cos ^{2} x$ $\sec ^{2} x-1=\tan ^{2} x$
$\cos ^{2} x-1=-\sin ^{2} x$

### 5.1 Using Fundamental Identities.notebook



Factoring and Foiling to Simplify
4. $\frac{\sin ^{2} \beta+1}{\sin \beta-1}$ No! $\frac{\sin ^{2} \beta-1}{\sin \beta-1} \Rightarrow$ difference of squares
$\sin \beta=x$
$\frac{(\sin \beta-1)(\sin \beta+1)}{\sin \beta-1}$ $\sin \beta=x \quad \frac{x^{2}-1}{x-1}=\frac{(x-x)(x+1)}{x-1}$ $=x+1$
5. $\sec ^{4} x-\tan ^{4} x \quad x^{4}-y^{4}=\left(x^{2}-y^{2}\right)\left(x^{2}+y^{2}\right)$ $\left(\sec ^{2} x^{\prime} \tan ^{2} x\right)\left(\sec ^{2} x+\tan ^{2} x\right)$

$$
\sec ^{2} x+\tan ^{2} x
$$

### 5.1 Using Fundamental Identities.notebook

$$
\begin{aligned}
& \text { 6. }(5-5 \sin x)(5+5 \sin x) \\
& 25+25 \sin x-25 \sin x-25 \sin ^{2} x \\
& 25-25 \sin ^{2} x \\
& 25\left(1-\sin ^{2} x\right) \\
& 25 \cos ^{2} x \\
& \text { 7. } \frac{1}{\sec x-1}-\frac{1}{\sec x+1} \\
& \frac{(\sec x+1)-(\sec x-1)}{(\sec x-1)(\sec x+1)} \\
& \frac{\sec x+1-\sec x+1}{\sec ^{2} x-\sec x+\sec x-1} \\
& \frac{2}{\sec ^{2} x-1} \\
& \frac{2}{\tan ^{2} x} \\
& 2 \cot ^{2} x
\end{aligned}
$$

