GO COUGARS!

p 379 Homework Questions

In Exercises 27–44, use the fundamental identities to simplify the expression. There is more than one correct form of each answer.

37. $\cos\left(\frac{\pi}{2} - x\right)\sec x$

39. $\frac{\cos^2 y}{1 - \sin y}$ $= \frac{1-\sin^2 y}{1-\sin y} = \frac{(1-\sin y)(1+\sin y)}{1-\sin y}$

In Exercises 61–64, perform the addition or subtraction and use the fundamental identities to simplify. There is more than one correct form of each answer.

61. $\frac{1}{1+\cos x} + \frac{1}{1-\cos x}$

63. $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x}$ $= \frac{\cos^2 x + (1+\sin x)^2}{\cos x(1+\sin x)} = \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos(1+\sin x)}$

In Exercises 65–68, rewrite the expression so that it is not in fractional form. There is more than one correct form of each answer.

65. $\frac{\sin^2 y}{1-\cos y}$ $\frac{2+2\sin x}{\cos x(1+\sin x)} = \frac{2(1+\sin x)}{\cos x(1+\sin x)} = 2\sec x$

67. $\frac{3}{\sec x - \tan x} \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \frac{3\sec x + 3\tan x}{\sec^2 x - \tan^2 y} = 3\sec x + 3\tan x$

In Exercises 73–76, use a graphing utility to determine which of the six trigonometric functions is equal to the expression. Verify your answer algebraically.

73. $\cos x \cot x + \sin x$

75. $\frac{1}{\sin x}\left(\frac{1}{\cos x} - \frac{\cos x}{1}\right)$ $= \frac{1}{\sin x}\left(\frac{1-\cos^2 x}{\cos x}\right) = \frac{1}{\sin x}\left(\frac{\sin x}{\cos x}\right) = \tan x$

In Exercises 77–82, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

79. $\sqrt{x^2 - 9}, \quad x = 3\sec\theta$ $\sqrt{(3\sec\theta)^2 - 9} = \sqrt{9\sec^2\theta - 9} = \sqrt{9(\sec^2\theta - 1)}$ $= 3\sqrt{\tan^2\theta}$ $= 3\tan\theta$

81. $\sqrt{x^2 + 25}, \quad x = 5\tan\theta$

In Exercises 91–94, rewrite the expression as a single logarithm and simplify the result.

91. $\ln|\cos x| - \ln|\sin x|$ $\ln\left|\frac{\cos x}{\sin x}\right| = \ln|\cot x|$

93. $\ln|\cot t| + \ln(1 + \tan^2 t)$

$\ln|\cot t(1+\tan^2 t)|$

$\ln\left|\frac{\cos t}{\sin t}(\sec^2 t)\right|$

$\ln\left|\frac{\cos t}{\sin t}\cdot\frac{1}{\cos^2 t}\right|$

$\ln|\csc t \sec t|$

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5.2 Verifying Trigonometric Identities

to verify means to prove one side is equal to the other side

- work with one side only
- you cannot use algebraic operations on both sides!
- look for ways to simplify including
  - > Pythagorean identity pairs
  - > reciprocals
  - > factoring
  - > multiplying
  - > common denominators
  - > putting into terms of sine and cosine

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$$\frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sec^2\theta}=1$$

LHS $\frac{1}{\cos^2\theta\sec^2\theta}=1$ identity

reciprocals

$\frac{\cos^2\theta}{\cos^2\theta}$

$1=1$ ✓

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$$\frac{1}{1-\cos\beta}+\frac{1}{1+\cos\beta}=2\csc^2\beta$$

LHS $\frac{1+\cos\beta+1-\cos\beta}{(1-\cos\beta)(1+\cos\beta)}$ Common denominator

Simplify / multiply

$\frac{2}{1-\cos^2\beta}$

$\frac{2}{\sin^2\beta}$ identity

$2\csc^2\beta=2\csc^2\beta$ ✓ reciprocal

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$(\sec^2 x - 1)(\sin^2 x - 1) = -\sin^2 x$

LHS

$(\tan^2 x)(-\cos^2 x)$

$\frac{\sin^2 x}{\cos^2 x}\left(\frac{-\cos^2 x}{1}\right)$

$-\sin^2 x$

identities

put in terms of sine/cosine

simplify

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$\csc x - \sin x = \cos x \cot x$

LHS

$\frac{1}{\sin x} - \frac{\sin x}{1}$

$\frac{1-\sin^2 x}{\sin x}$

$\frac{\cos^2 x}{\sin x}$

$\cos x\left(\frac{\cos x}{\sin x}\right)$

$\cos x \cot x = \cos x \cot x$ ✓

(LCD of minus)

reciprocal

common denominator

identity

split

simplify

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$$\csc\beta + \cot\beta = \frac{\sin\beta}{1-\cos\beta}$$

LHS

$\frac{1}{\sin\beta} + \frac{\cos\beta}{\sin\beta}$

$\frac{1+\cos\beta}{\sin\beta} \cdot \frac{1-\cos\beta}{1-\cos\beta}$

$\frac{1-\cos^2\beta}{\sin\beta(1-\cos\beta)}$

$\frac{\sin^2\beta}{\sin\beta(1-\cos\beta)}$

$\frac{\sin\beta}{1-\cos\beta} = \frac{\sin\beta}{1-\cos\beta}$ ✓

RHS

$\frac{\sin\beta}{1-\cos\beta} \cdot \frac{1+\cos\beta}{1+\cos\beta}$

$\frac{\sin\beta(1+\cos\beta)}{1-\cos^2\beta}$

$\frac{\sin\beta(1+\cos\beta)}{\sin^2\beta}$

$\frac{1+\cos\beta}{\sin\beta}$

$\frac{1}{\sin\beta} + \frac{\cos\beta}{\sin\beta}$

$\csc\beta + \cot\beta$ ✓

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$$\frac{\tan^2\theta}{1+\sec\theta} = \frac{1-\cos\theta}{\cos\theta}$$

LHS

$\frac{\sec^2\theta - 1}{1+\sec\theta}$

$\frac{(\sec\theta-1)(\sec\theta+1)}{1+\sec\theta}$

$\sec\theta - 1$

$\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}$

$\frac{1-\cos\theta}{\cos\theta}$

RHS $\frac{1-\cos\theta}{\cos\theta}$

$\frac{1}{\cos\theta} - \frac{\cos\theta}{\cos\theta}$

$\sec\theta - 1$

$\frac{(\sec\theta-1)(\sec\theta+1)}{\sec\theta+1}$

$\frac{\sec^2\theta - 1}{\sec\theta+1}$

$\frac{\tan^2\theta}{\sec\theta+1}$ ✓

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$$\cos^3 x \sin^4 x = (\sin^4 x - \sin^6 x)\cos x$$

RHS $= \sin^4 x (1 - \sin^2 x)\cos x$

$\sin^4 x \cos^2 x \cdot \cos x$

$\sin^4 x \cos^3 x$

LHS

$\sin^4 x \cos^2 x \cos x$

$\sin^4 x \cos x (1 - \sin^2 x)$

$\cos x (\sin^4 x - \sin^6 x)$

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$$\tan^3 x = \tan x \sec^2 x - \tan x$$

RHS $\tan x (\sec^2 x - 1)$

$\tan x (\tan^2 x)$

$\tan^3 x$ ✓

LSH $\tan^2 x \tan x$

$(\sec^2 x - 1)\tan x$

$\sec^2 x \tan x - \tan x$ ✓

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$\cot^4 x \csc x = \cot^2 x(\csc^3 x - \csc x)$

RHS $= \cot^2 x \cdot \csc x (\csc^2 x - 1)$

$= \cot^2 x \cdot \csc x \cdot \cot^2 x$

$= \cot^4 x \csc x$ ✓

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HOMEWORK

p 387 5-11 odd, 15-29 odd, 35, 37, (39, 43 algebraically), 51, 53

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