5.4 Sum and Difference Formulas.notebook

p 376 **Homework Questions**

49. $2\sin^2 x + 3\sin x + 1 = 0$
50. $2\sec^2 x + \tan^2 x - 3 = 0$
51. $4\sin^2 x = 2\cos x + 1$
52. $\csc^2 x = 3\csc x + 4$

53. $\csc x + \cot x = 1$
54. $4\sin x = \cos x - 2$
55. $\dfrac{\cos x \cot x}{1 - \sin x} = 3$

$\overline{\cos x (1 + \sin x)}$

56. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} = 4$

In Exercises 57–60, (a) use a graphing utility to graph each function in the interval $[0, 2\pi)$, (b) write an equation whose solutions are the points of intersection of the graphs, and (c) use the *intersect* feature of the graphing utility to find the points of intersection (to four decimal places).

57. $y = \sin 2x, \quad y = x^2 - 2x$
58. $y = \cos x, \quad y = x + x^2$
59. $y = \sin^2 x, \quad y = e^x - 4x$
60. $y = \cos^2 x, \quad y = e^{-x} + x - 1$

In Exercises 61–72, solve the multiple-angle equation.

61. $\cos \dfrac{x}{4} = 0$ 62. $\sin \dfrac{x}{2} = 0$
63. $\sin 4x = 1$ 64. $\cos 2x = -1$
65. $\sin 2x = -\dfrac{\sqrt{3}}{2}$ 66. $\sec 4x = 2$
67. $2\sin^2 2x = 1$ 68. $\tan^2 3x = 3$
69. $\tan 3x(\tan x - 1) = 0$ 70. $\cos 2x(2\cos x + 1) = 0$
71. $\cos \dfrac{x}{2} = \dfrac{\sqrt{2}}{2}$ 72. $\tan \dfrac{x}{3} = 1$

In Exercises 77–84, use a graphing utility to approximate the solutions of the equation in the interval $[0, 2\pi)$.

77. $2\cos x - \sin x = 0$
78. $2\sin x + \cos x = 0$
79. $x \tan x - 1 = 0$
80. $2x \sin x - 2 = 0$
81. $\sec^2 x + 0.5 \tan x = 1$
82. $\csc^2 x + 0.5 \cot x = 5$
83. $12\sin^2 x - 13 \sin x + 3 = 0$
84. $3\tan^2 x + 4 \tan x - 4 = 0$

In Exercises 85–88, use a graphing utility to approximate the solutions (to three decimal places) of the equation in the given interval.

85. $3\tan^2 x + 5 \tan x - 4 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
86. $\cos^2 x - 2 \cos x - 1 = 0, \quad [0, \pi]$
87. $4\cos^2 x - 2 \sin x + 1 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$
88. $2\sec^2 x + \tan x - 6 = 0, \quad \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

Handwritten (right side):

$\sin 4x = 1$

$4x = \dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}, \dfrac{13\pi}{2}$

$= \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$

$\cos \dfrac{x}{2} = \dfrac{1}{\sqrt{2}}$

$12\left(\sin(x)\right)^2 - 13 \sin x + 3$

Feb 2-9:51 PM

5.4 Sum and Difference Formulas

find exact values given an angle
condense
verify identities
find exact values given ratios

Feb 26-7:12 AM

1

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\Rightarrow \tan(u{+}v) = \frac{\sin(u{+}v)}{\cos(u{+}v)}$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

$$\cancel{\sin u + \sin v}$$

$$\sin 106 \cos 16 - \cos 106 \sin 16$$

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'distributing the trig function' is not a thing!

sin, cos, tan, etc is not a constant,

they are relationships based on the sides and angles of a triangle

so... $\sin(30° + 60°) = \sin 90°$

$$= 1$$

but $\sin 30° + \sin 60° \neq \sin 90°$

$$\frac{1}{2} + \frac{\sqrt{3}}{2} \neq 1$$

Mar 11-6:21 AM

The formulas are used to find the <u>exact</u> value w/o calculator.

1. $\cos(105°) =$ $\cos(45+60)$

$\cos 45 \cos 60 - \sin 45 \sin 60$

$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$\frac{1-\sqrt{3}}{2\sqrt{2}}$

book:

$\frac{\sqrt{2}-\sqrt{6}}{4}$

2. $\sin\dfrac{5\pi}{12} =$ $\sin\left(\frac{3\pi}{12} + \frac{2\pi}{12}\right)$

$\sin\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$

$\frac{\sqrt{3}+1}{2\sqrt{2}}$

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3. $\sin 106° \cos 16° - \cos 106° \sin 16°$

$\sin(106 - 16)$

$\sin 90$

1

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3

These formulas can also be used to prove identities.

4. $\cos(\dfrac{\pi}{2} - x) = \sin x$
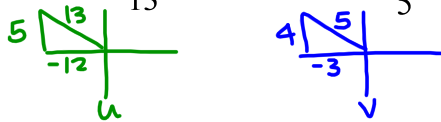
$\cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$0\cdot\cos x + 1\cdot\sin x$

$\sin x = \sin x$ ✓

Feb 16-1:26 PM

And....to evaluate trig angle sums and differences.

5. Given $\sin u = \dfrac{5}{13}$ and $\cos v = -\dfrac{3}{5}$ in QII find:

$\sin(u-v) = (\sin u)(\cos v) - (\cos u)(\sin v)$

$\left(\dfrac{5}{13}\right)\left(-\dfrac{3}{5}\right) - \left(-\dfrac{12}{13}\right)\left(\dfrac{4}{5}\right)$

$\dfrac{-15}{65} - \left(\dfrac{-48}{65}\right)$

$\dfrac{-15}{65} + \dfrac{48}{65}$

$\dfrac{33}{65}$

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4

HOMEWORK

p 384 3, 7, 15, 21, 23-29 odd,

35-41 odd, 47, 52, 63

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GO COUGARS!

Homework Questions

WB p 126 answers

1a. $\frac{\pi}{3}, \frac{2\pi}{3}$ b. 0 g. $\frac{5\pi}{6}, \frac{7\pi}{6}$ k. $\frac{\pi}{2}$ l. $\frac{\pi}{3}, \frac{5\pi}{3}$

2a. $1.744, 4.539$ b. $0.588, 2.554$

3c. $3.481, 5.943$ g. $\frac{3\pi}{2}$

4. $1.823, 4.460$ 6. $\frac{\pi}{2}, \frac{3\pi}{2}$ 10. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

12. $\frac{7\pi}{24} \frac{11\pi}{24} \frac{19\pi}{24} \frac{23\pi}{24} \frac{31\pi}{24} \frac{35\pi}{24} \frac{43\pi}{24} \frac{47\pi}{24}$ 13. $0 \frac{2\pi}{3} \frac{4\pi}{3}$

15. $\frac{\pi}{6}, \frac{5\pi}{6} \frac{7\pi}{6} \frac{11\pi}{6}$ 16. $\frac{7\pi}{18} \frac{11\pi}{18} \frac{19\pi}{18} \frac{23\pi}{18} \frac{31\pi}{18} \frac{35\pi}{18}$

17. $\frac{\pi}{4} \frac{3\pi}{4} \frac{5\pi}{4} \frac{7\pi}{4}$ 20. $0 \ \pi \ \frac{7\pi}{6}, \frac{11\pi}{6}$

21. $0 \ \frac{\pi}{3} \ \frac{7\pi}{18} \ \frac{11\pi}{18} \ \frac{2\pi}{3} \ \pi \ \frac{19\pi}{18} \ \frac{23\pi}{18} \ \frac{4\pi}{3} \ \frac{5\pi}{3} \ \frac{31\pi}{18} \ \frac{35\pi}{18}$

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