7.1 In Exercises 1–6, name the trigonometric function that is equivalent to the expression.

1. $\dfrac{1}{\cos x}$
2. $\dfrac{1}{\sin x}$
3. $\dfrac{1}{\sec x}$
4. $\dfrac{1}{\tan x}$
5. $\dfrac{\cos x}{\sin x}$
6. $\sqrt{1 + \tan^2 x}$

In Exercises 7–10, use the given values and trigonometric identities to evaluate (if possible) the other trigonometric functions of the angle.

7. $\sin x = \frac{3}{5}$, $\cos x = \frac{4}{5}$
8. $\tan \theta = \dfrac{2}{3}$, $\sec \theta = \dfrac{\sqrt{13}}{3}$
9. $\sin\left(\dfrac{\pi}{2} - x\right) = \dfrac{\sqrt{2}}{2}$, $\sin x = -\dfrac{\sqrt{2}}{2}$
10. $\csc\left(\dfrac{\pi}{2} - \theta\right) = 9$, $\sin \theta = \dfrac{4\sqrt{5}}{9}$

In Exercises 11–22, use the fundamental trigonometric identities to simplify the trigonometric expression.

11. $\dfrac{1}{\cot^2 x + 1} = \sin^2 x$
12. $\dfrac{\tan \theta}{1 - \cos^2 \theta}$
13. $\tan^2 x(\csc^2 x - 1) = 1$
14. $\cot^2 x(\sin^2 x)$
15. $\dfrac{\sin\left(\dfrac{\pi}{2} - \theta\right)}{\sin \theta}$
16. $\dfrac{\cot\left(\dfrac{\pi}{2} - u\right)}{\cos u}$
17. $\cos^2 x + \cos^2 x \cot^2 x = \cot^2 x$
18. $\tan^2 \theta \csc^2 \theta - \tan^2 \theta$
19. $(\tan x + 1)^2 \cos x$
20. $(\sec x - \tan x)^2$
21. $\dfrac{1}{\csc \theta + 1} - \dfrac{1}{\csc \theta - 1} = -2\tan^2\theta$
22. $\dfrac{\cos^2 x}{1 - \sin x}$

23. ***Rate of Change*** The rate of change of the function $f(x) = \csc x - \cot x$ is the expression $\csc^2 x - \csc x \cot x$. Show that this expression can also be written as

$$\frac{(1 - \cos x)}{\sin^2 x}.$$

24. ***Rate of Change*** The rate of change of the function $f(x) = 2\sqrt{\sin x}$ is the expression $\sin^{-1/2} x \cos x$. Show that this expression can also be written as $\cot x\sqrt{\sin x}$.

7.2 In Exercises 25–32, verify the identity.

25. $\cos x(\tan^2 x + 1) = \sec x$
26. $\sec^2 x \cot x - \cot x = \tan x$
27. $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$
28. $\cot\left(\dfrac{\pi}{2} - x\right) = \tan x$
29. $\dfrac{1}{\tan \theta \csc \theta} = \cos \theta$
30. $\dfrac{1}{\tan x \csc x \sin x} = \cot x$
31. $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x) \sin x$
32. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x) \cos x$

7.3 In Exercises 33–38, solve the equation.

33. $\sin x = \sqrt{3} - \sin x$
34. $4 \cos \theta = 1 + 2 \cos \theta$
35. $3\sqrt{3} \tan u = 3$
36. $\frac{1}{2} \sec x - 1 = 0$
37. $3 \csc^2 x = 4$
38. $4 \tan^2 u - 1 = \tan^2 u$

In Exercises 39–46, find all solutions of the equation in the interval $[0, 2\pi)$.

39. $2\cos^2 x - \cos x = 1$
40. $2\sin^2 x - 3 \sin x = -1$
41. $\cos^2 x + \sin x = 1$
42. $\sin^2 x + 2 \cos x = 2$
43. $2 \sin 2x - \sqrt{2} = 0$
44. $\sqrt{3} \tan 3x = 0$
45. $\cos 4x(\cos x - 1) = 0$
46. $3 \csc^2 5x = -4$

In Exercises 47–50, use inverse functions where needed to find all solutions of the equation in the interval $[0, 2\pi)$.

47. $\sin^2 x - 2 \sin x = 0$
48. $2\cos^2 x + 3 \cos x = 0$
49. $\tan^2 \theta + \tan \theta - 12 = 0$
50. $\sec^2 x + 6 \tan x + 4 = 0$

7.4 **In Exercises 51–54, find the exact values of the sine, cosine, and tangent of the angle by using a sum or difference formula.**

51. $285°$
52. $345°$
53. $\frac{25\pi}{12}$
54. $\frac{19\pi}{12}$

In Exercises 55–58, write the expression as the sine, cosine, or tangent of an angle.

55. $\sin 60° \cos 45° - \cos 60° \sin 45°$
56. $\cos 45° \cos 120° - \sin 45° \sin 120°$
57. $\dfrac{\tan 25° + \tan 10°}{1 - \tan 25° \tan 10°}$
58. $\dfrac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

In Exercises 59–64, find the exact value of the trigonometric function given that $\sin u = \frac{3}{4}$, $\cos v = -\frac{5}{13}$, and u and v are in Quadrant II.

59. $\sin(u + v)$
60. $\tan(u + v)$
61. $\cos(u - v)$
62. $\sin(u - v)$
63. $\cos(u + v)$
64. $\tan(u - v)$

In Exercises 65–68, find all solutions of the equation in the interval $[0, 2\pi)$.

65. $\sin\left(x + \frac{\pi}{4}\right) - \sin\left(x - \frac{\pi}{4}\right) = 1$

66. $\cos\left(x + \frac{\pi}{6}\right) - \cos\left(x - \frac{\pi}{6}\right) = 1$

67. $\sin\left(x + \frac{\pi}{2}\right) - \sin\left(x - \frac{\pi}{2}\right) = \sqrt{3}$

68. $\cos\left(x + \frac{3\pi}{4}\right) - \cos\left(x - \frac{3\pi}{4}\right) = 0$

7.5 **In Exercises 69 and 70, use double-angle formulas to verify the identity algebraically and use a graphing utility to confirm it graphically.**

69. $\sin 4x = 8 \cos^3 x \sin x - 4 \cos x \sin x$

70. $\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

In Exercises 71 and 72, find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

71. $\sin u = -\frac{4}{5}$, $\pi < u < \frac{3\pi}{2}$ (Q III)

72. $\cos u = -\frac{2}{\sqrt{5}}$, $\frac{\pi}{2} < u < \pi$ (Q II)

73. ***Projectile Motion*** A baseball leaves the hand of the person at first base at an angle of θ with the horizontal and at an initial velocity of $v_0 = 80$ feet per second. The ball is caught by the person at second base 100 feet away. Find θ if the range r of a projectile is

$$r = \frac{1}{32} v_0^2 \sin 2\theta.$$

74. ***Projectile Motion*** Use the equation in Exercise 73 to find θ when a golf ball is hit at an initial velocity of 50 feet per second and lands 77 feet away.

In Exercises 75–78, use the power-reducing formulas to rewrite the expression in terms of the first power of the cosine.

75. $\tan^2 2x$
76. $\cos^2 3x$
77. $\sin^2 x \tan^2 x$
78. $\cos^2 x \tan^2 x$

In Exercises 79–82, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

79. $-75°$
80. $15°$
81. $\frac{19\pi}{12}$
82. $-\frac{17\pi}{12}$

In Exercises 83 and 84, use the half-angle formulas to simplify the expression.

83. $-\sqrt{\dfrac{1 + \cos 10x}{2}}$
84. $\dfrac{\sin 6x}{1 + \cos 6x}$

85. Find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ for $\sin u = \frac{3}{5}$, $0 < u < \frac{\pi}{2}$.

86. ***Geometry*** A trough for feeding cattle is 4 meters long and its cross sections are isosceles triangles with the two equal sides being $\frac{1}{2}$ meter (see figure on page 589). The angle between the two sides is θ.

(a) Write the trough's volume as a function of $\frac{\theta}{2}$.

(b) Write the volume of the trough as a function of θ and determine the value of θ such that the volume is maximum.

Review

11) $\frac{1}{\cot^2 x+1} = \sin^2 x$

$\frac{1}{\csc^2 x} = \sin^2 x$

$\sin^2 x = \sin^2 x$

13) $\tan^2 x(\csc^2 x-1)=1$

$\tan^2 x(\cot^2 x)=1$

$1=1$

17) $\cos^2 x + \cos^2 x \cot^2 x = \cot^2 x$

$\cos^2 x(1+\cot^2 x)$

$\cos^2 x(\csc^2 x)$

$\frac{\cos^2 x}{\sin^2 x}$

$\cot^2 x$

21) $\frac{1}{\csc\theta+1} - \frac{1}{\csc\theta-1} = -2\tan^2\theta$

$\frac{(\csc\theta-1)-(\csc\theta+1)}{(\csc\theta+1)(\csc\theta-1)}$

$\frac{-2}{\csc^2\theta-1}$

$\frac{-2}{\cot^2\theta}$

$-2\tan^2\theta$

25) $\cos x(\tan^2 x+1)=\sec x$

$\cos x(\sec^2 x)$

$\sec x = \sec x$

26) $\sec^2 x\cot x - \cot x = \tan x$

$\cot x(\sec^2 x-1)=$

$\cot x(\tan^2 x)$

$\tan x$

29) $\frac{1}{\tan x\csc x} = \cos x$

$\cot x\sin x$

$\frac{\cos x}{\sin x}\cdot\sin x$

$\cos x$

33) $\sin x = \sqrt{3} - \sin x$

$2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}+2k\pi,\ \frac{2\pi}{3}+2k\pi$

34) $4\cos\theta = 1+2\cos\theta$

$2\cos\theta = 1$

$\cos\theta = \frac{1}{2}$

$\theta = \frac{\pi}{3}+2k\pi,\ \frac{5\pi}{3}+2k\pi$

37) $3\csc^2 x = 4$

$\csc^2 x = \frac{4}{3}$

$\csc x = \pm\frac{2}{\sqrt{3}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}+k\pi,\ \frac{2\pi}{3}+k\pi$

38) $4\tan^2 u - 1 = \tan^2 u$

$3\tan^2 u = 1$

$\tan u = \pm\frac{1}{\sqrt{3}}$ $\quad u = \frac{\pi}{6}+k\pi,\ \frac{5\pi}{6}+k\pi$

39) $2\cos^2 x - \cos x = 1$

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ | $\cos x = 1$

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$ | $x = 0$

41) $\cos 2x + \sin x = 1$

$1 - 2\sin^2 x + \sin x = 1$

$-2\sin^2 x + \sin x = 0$

$\sin x(-2\sin x + 1) = 0$

$\sin x = 0$ | $\sin x = \frac{1}{2}$

$x = 0, \pi$ | $x = \frac{\pi}{6}, \frac{5\pi}{6}$

43) $2\sin 2x - \sqrt{2} = 0$

$2\sin 2x = \sqrt{2}$

$\sin 2x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

44) $\sqrt{3}\tan 3x = 0$

$\tan 3x = 0$

$3x = \underbrace{0, \pi}, \underbrace{2\pi, 3\pi}, \underbrace{4\pi, 5\pi}$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

51) $\sin 285 = \sin(45 + 240)$

$= \sin 45 \cos 240 + \cos 45 \sin 240$

$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{-\sqrt{3}}{2}\right)$

$= -1 - \sqrt{3}$

51) cont...

$\cos(45+240) = \cos 45 \cos 240 - \sin 45 \sin 240$

$= \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(-\frac{\sqrt{3}}{2}\right)$

$= -\frac{1+\sqrt{3}}{2\sqrt{2}}$

$\tan(45+240) = \frac{-1-\sqrt{3}}{-1+\sqrt{3}}$

53) $\sin\left(\frac{25\pi}{12}\right) = \sin\left(\frac{\pi}{12}\right) = \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$\sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \cos\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$\frac{\sqrt{3}-1}{2\sqrt{2}}$

$\cos\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

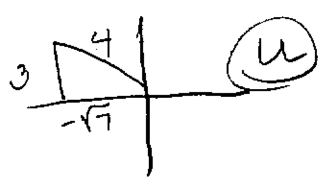
$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

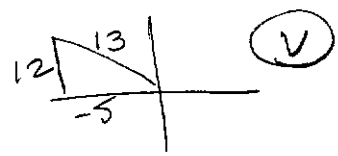
$\tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

55) $\sin 25$

56) $\cos 165$

$\sin u = \frac{3}{4}$ $\qquad \cos v = \frac{-5}{13}$ $\quad$ Q II





57) $\sin(u+v) = \sin u \cos v + \cos u \sin v$

$= \left(\frac{3}{4}\right)\left(-\frac{5}{13}\right) + \left(-\frac{\sqrt{7}}{4}\right)\left(\frac{+12}{13}\right)$

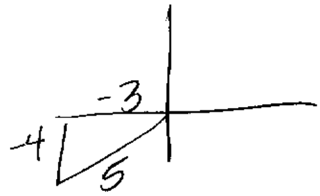
$= \frac{-15-2\sqrt{7}}{52}$

61) $\cos(u-v) = \cos u \cos v + \sin u \sin v$

$= \left(-\frac{\sqrt{7}}{4}\right)\left(-\frac{5}{13}\right) + \left(\frac{3}{4}\right)\left(\frac{12}{13}\right)$

$= \frac{5\sqrt{7}+36}{4\sqrt{7}}$

71) $\sin u = -\frac{4}{5}$ Q III



$\sin 2u = 2\sin u \cos u$

$= 2\left(-\frac{4}{5}\right)\left(-\frac{3}{5}\right)$

$= \frac{24}{25}$

$\cos 2u = \cos^2 u - \sin^2 u$

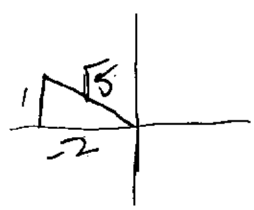
$= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2$

$= \frac{9}{25} - \frac{16}{25}$

$= -\frac{7}{25}$

$\tan 2u = -\frac{24}{7}$

72) $\cos u = -\frac{2}{\sqrt{5}}$ Q II



$\sin 2u = 2\sin u \cos u$

$= 2\left(\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right)$

$= \frac{-4}{5}$

$\cos 2u = \cos^2 u - \sin^2 u$

$= \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{3}{5}$

75) $\tan^2 2x = \frac{1-\cos 4x}{1+\cos 4x}$

77) $\sin^2 x \tan^2 x$

$\left(\frac{1-\cos 2x}{2}\right)\left(\frac{1-\cos 2x}{1+\cos 2x}\right)$

$\frac{1-2\cos 2x+\cos^2 2x}{2+2\cos 2x}$

$\frac{1-2\cos 2x+\left(\frac{1+\cos 4x}{2}\right)}{2+2\cos 2x}$

$\frac{2-4\cos 2x+1+\cos 4x}{4+4\cos 2x}$

$\frac{3-4\cos 2x+\cos 4x}{4+4\cos 2x}$