

# Multiple-Angle Problems Key

1)  $2\sin^2 2x = 1$

$\sin^2 2x = \frac{1}{2}$

$\sin 2x = \pm \frac{\sqrt{2}}{2}$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$\frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

$\frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

2)  $\tan^2 3x = 3$

$\tan 3x = \pm \sqrt{3}$

$3x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}$

$\frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}$

$x = \frac{\pi}{9}, \frac{2\pi}{9}, \frac{4\pi}{9}, \frac{5\pi}{9}$

$\frac{7\pi}{9}, \frac{8\pi}{9}, \frac{10\pi}{9}, \frac{11\pi}{9}$

$\frac{13\pi}{9}, \frac{14\pi}{9}, \frac{16\pi}{9}, \frac{17\pi}{9}$

3)  $\tan 3x(\tan x - 1) = 0$

$\tan 3x = 0$      $\tan x = 1$

$3x = 0, \pi$

$2\pi, 3\pi$

$4\pi, 5\pi$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi$

$\frac{4\pi}{3}, \frac{5\pi}{3}$

4)  $\cos 2x(2\cos x + 1) = 0$

$\cos 2x = 0$      $\cos x = -\frac{1}{2}$

$2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\dots \frac{5\pi}{2}, \frac{7\pi}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

5)  $\sin 4x - 2\sin 2x = 0$

$2\sin 2x \cos 2x - 2\sin 2x = 0$

$2\sin 2x(\cos 2x - 1) = 0$

$\sin 2x = 0$      $\cos 2x = 1$

$2x = 0, \pi, 2\pi, 3\pi$

$2x = 0, 2\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$x = 0, \pi$

6)  $\cos 2x \sin x + \sin x = 0$

$\sin x(\cos 2x + 1) = 0$

$\sin x = 0$      $\cos 2x = -1$

$x = 0, \pi$

$2x = \pi, 3\pi$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

7)  $2\sin 2x - \sqrt{2} = 0$

$\sin 2x = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

8)  $\sqrt{3}\tan 3x = 0$

$\tan 3x = 0$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

9)  $\tan 3x = 1$

$3x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \frac{17\pi}{4}, \frac{21\pi}{4}$

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{9\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{21\pi}{12}$

$$10) \sec 4x = 2$$

$$\cos 4x = \frac{1}{2}$$

$$4x = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{23\pi}{3}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$$

$$11) \tan^2\left(\frac{x}{2}\right) = \frac{1}{3}$$

$$\tan \frac{x}{2} = \pm \frac{1}{\sqrt{3}}$$

$$\frac{x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$12) \sin x \cos\left(\frac{x}{2}\right) = \cos \frac{x}{2}$$

$$\sin x \cos \frac{x}{2} - \cos \frac{x}{2} = 0$$

$$\cos \frac{x}{2} (\sin x - 1) = 0$$

$$\cos \frac{x}{2} = 0 \quad \sin x = 1$$

$$\frac{x}{2} = \frac{\pi}{2} \quad x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} \quad x = \frac{\pi}{2}$$