

Chapter 5 Study Guide

$$\begin{aligned} 1) \cos \frac{7\pi}{12} &= \cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) \\ &= \cos \frac{\pi}{4} \cos \frac{\pi}{3} - \sin \frac{\pi}{4} \sin \frac{\pi}{3} \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{1 - \sqrt{3}}{2\sqrt{2}} \end{aligned}$$

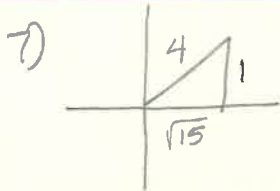
$$\begin{aligned} 2) \sin \frac{7\pi}{12} &= \sin\left(\frac{7\pi}{6} + \frac{\pi}{4}\right) \\ &= \sin \frac{7\pi}{6} \cos \frac{\pi}{4} + \cos \frac{7\pi}{6} \sin \frac{\pi}{4} \\ &= \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) \\ &= \frac{-1 - \sqrt{3}}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} 3) \cos(-15)^\circ &= \cos(45 - 60) \\ &= \cos 45 \cos 60 + \sin 45 \sin 60 \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) \\ &= \frac{1 + \sqrt{3}}{2\sqrt{2}} \end{aligned}$$

$$\begin{aligned} 4) \sin 10^\circ \cos 80^\circ + \cos 10^\circ \sin 80^\circ &= \sin(10 + 80) \\ &= \sin 90 \\ &= 1 \end{aligned}$$

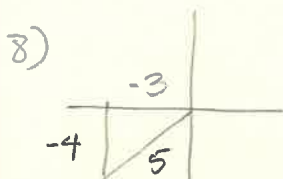
$$\begin{aligned} 5) \cos \frac{2\pi}{3} \cos \frac{\pi}{3} - \sin \frac{2\pi}{3} \sin \frac{\pi}{3} &= \cos\left(\frac{2\pi}{3} + \frac{\pi}{3}\right) \\ &= \cos \frac{\pi}{3} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} 6) \sin 60 \cos 15 + \cos 60 \sin 15 &= \sin 75 \\ \sin 75 &= \sin(45 + 30) \\ &= \sin 45 \cos 30 + \cos 45 \sin 30 \\ &= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) \\ &= \frac{\sqrt{3} + 1}{2\sqrt{2}} \end{aligned}$$



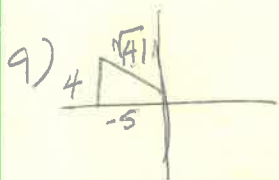
$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ &= 2 \left(\frac{4}{4}\right) \left(\frac{\sqrt{15}}{4}\right) \\ &= \frac{2\sqrt{15}}{16} \\ &= \frac{\sqrt{15}}{8}\end{aligned}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= \left(\frac{\sqrt{15}}{4}\right)^2 - \left(\frac{4}{4}\right)^2 \\ &= \frac{15}{16} - \frac{16}{16} \\ &= \frac{14}{16} = \frac{7}{8}\end{aligned}$$



$$\begin{aligned}\sin 2y &= 2 \sin y \cos y \\ &= 2 \left(-\frac{4}{5}\right) \left(-\frac{3}{5}\right) \\ &= \frac{24}{25}\end{aligned}$$

$$\begin{aligned}\cos 2y &= \cos^2 y - \sin^2 y \\ &= \left(-\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 \\ &= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{25}\end{aligned}$$



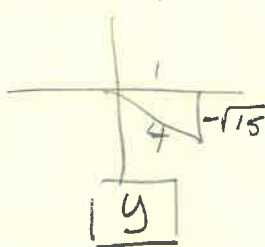
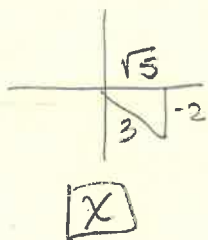
$$\begin{aligned}\sin 2x &= 2 \sin x \cos x \\ &= 2 \left(\frac{4}{41}\right) \left(-\frac{5}{41}\right) \\ &= \frac{-40}{41}\end{aligned}$$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= \left(-\frac{5}{41}\right)^2 - \left(\frac{4}{41}\right)^2 \\ &= \frac{25}{41} - \frac{16}{41} \\ &= \frac{9}{41}\end{aligned}$$

$$\tan 2x = \frac{-\frac{40}{41}}{\frac{9}{41}} = -\frac{40}{9}$$

$$\sin x = -\frac{2}{3}$$

$$\cos y = \frac{1}{4}$$



$$\begin{aligned}10) \sin(x-y) &= \sin x \cos y - \cos x \sin y \\ &= \left(-\frac{2}{3}\right) \left(\frac{1}{4}\right) - \left(-\frac{\sqrt{5}}{3}\right) \left(-\frac{\sqrt{15}}{4}\right) \\ &= -\frac{2}{12} + \frac{\sqrt{75}}{12} \\ &= \frac{-2 + 5\sqrt{3}}{12}\end{aligned}$$

$$\begin{aligned}
 11) \cos(\theta - \pi) &= \cos\theta \cos\pi + \sin\theta \sin\pi \\
 &= \left(\frac{1}{4}\right)\left(\frac{\sqrt{5}}{3}\right) + \left(-\frac{\sqrt{15}}{4}\right)\left(-\frac{2}{3}\right) \\
 &= \frac{\sqrt{5}}{12} + \frac{2\sqrt{15}}{12} \\
 &= \frac{\sqrt{5} + 2\sqrt{15}}{12}
 \end{aligned}$$

$$\begin{aligned}
 12) \tan(x+y) &= \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} \\
 &= \frac{\left(\frac{2}{3}\right)\left(\frac{1}{4}\right) + \left(\frac{\sqrt{5}}{3}\right)\left(-\frac{\sqrt{15}}{4}\right)}{\left(\frac{\sqrt{5}}{3}\right)\left(\frac{1}{4}\right) - \left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{4}\right)} \\
 &= \frac{-2 - \sqrt{75}}{12} = \frac{-2 - 5\sqrt{3}}{15 - 2\sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 13) \frac{\tan\theta \csc\theta}{\sec\theta} &= 1 \\
 \frac{\frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} &= 1
 \end{aligned}$$

$$\begin{aligned}
 14) \frac{\cot^2\theta - 1}{1 + \cot^2\theta} &= 1 - 2\sin^2\theta \\
 \frac{\cot^2\theta - 1}{\csc^2\theta} &= 1 - 2\sin^2\theta
 \end{aligned}$$

$$\begin{aligned}
 \frac{\cot^2\theta}{\csc^2\theta} - \frac{1}{\csc^2\theta} &= 1 - 2\sin^2\theta \\
 \frac{\cos^2\theta}{\sin^2\theta} \cdot \frac{\sin^2\theta}{1} - \frac{1}{\csc^2\theta} &= 1 - 2\sin^2\theta \\
 \cos^2\theta - \sin^2\theta &= 1 - 2\sin^2\theta \\
 1 - \sin^2\theta - \sin^2\theta &= 1 - 2\sin^2\theta
 \end{aligned}$$

$$\begin{aligned}
 15) \frac{\sin^2\theta - \cot\theta \csc\theta}{\cot\theta \sin\theta} &= -\cos\theta \\
 \frac{\sin^2\theta - 1}{\cot\theta \sin\theta} &= -\cos\theta \\
 \frac{-\cos^2\theta}{\cot\theta \sin\theta} &= -\cos\theta \\
 \frac{-\cos^2\theta}{\frac{\cos\theta \sin\theta}{\sin\theta}} &= -\cos\theta \\
 -\cos\theta &= -\cos\theta
 \end{aligned}$$

$$\begin{aligned}
 16) \frac{\cos\theta}{\sec\theta - \tan\theta} &= 1 + \sin\theta \\
 \frac{\cos\theta}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} &= 1 + \sin\theta
 \end{aligned}$$

$$\begin{aligned}
 \cos\theta \cdot \frac{\cos\theta}{1 - \sin\theta} &= 1 + \sin\theta \\
 \frac{\cos^2\theta}{1 - \sin\theta} &= \frac{1 - \sin^2\theta}{1 - \sin\theta} = \frac{(1 - \sin\theta)(1 + \sin\theta)}{1 - \sin\theta}
 \end{aligned}$$

$$17) \tan \beta (\cot \beta + \tan \beta) = \sec^2 \beta$$

$$\frac{1 + \tan^2 \beta}{\sec^2 \beta}$$

$$18) \tan^2 \alpha \cos^2 \alpha = 1 - \cos^2 \alpha$$

$$\frac{\sin^2 \alpha}{\cos^2 \alpha} \cdot \frac{\cos^2 \alpha}{1}$$

$$\frac{\sin^2 \alpha}{1 - \cos^2 \alpha}$$

$$19) \csc x \sec x = \cot x + \tan x$$

$$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x}$$

$$\sec x \csc x$$

$$20) \cos^2 x + \tan^2 x \cos^2 x = 1$$

$$\cos^2 x (1 + \tan^2 x)$$

$$\cos^2 x (\sec^2 x)$$

$$1$$

$$21) \tan \theta \sin \theta = \frac{1 - \cos^2 \theta}{\cos \theta}$$

$$= \frac{\sin^2 \theta}{\cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta} \cdot \sin \theta$$

$$= \tan \theta \sin \theta$$

$$22) \sin x (\csc x - \sin x) = \cos^2 x$$

$$\frac{1 - \sin^2 x}{\cos^2 x}$$