

Sum - Diff & Multiple Angles

$$1) \sin 30 \cos 45 + \cos 30 \sin 45$$

$$\sin(30 + 45)$$

$$\sin 75^\circ$$

$$2) \cos 30 \cos 45 - \sin 30 \sin 45$$

$$\cos(30 + 45)$$

$$\cos 75^\circ$$

$$3) \sin \frac{\pi}{4} \cos \frac{2\pi}{3} - \cos \frac{\pi}{4} \sin \frac{2\pi}{3}$$

$$\sin\left(\frac{\pi}{4} - \frac{2\pi}{3}\right)$$

$$\sin\left(\frac{3\pi}{12} - \frac{8\pi}{12}\right)$$

$$\sin\left(-\frac{5\pi}{12}\right)$$

$$4) \cos \frac{7\pi}{6} \cos \frac{\pi}{3} + \sin \frac{7\pi}{6} \cos \frac{\pi}{3}$$

$$\cos\left(\frac{7\pi}{6} - \frac{\pi}{3}\right)$$

$$\cos\left(\frac{2\pi}{6} - \frac{2\pi}{6}\right)$$

$$\cos\left(\frac{5\pi}{6}\right)$$

$$5) \cos(75)$$

$$\cos(30 + 45)$$

$$\cos 30 \cos 45 - \sin 30 \sin 45$$

$$\frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{1}{2}$$

$$\frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$6) \cos 195$$

$$\cos(45 + 150)$$

$$\cos 45 \cos 150 - \sin 45 \sin 150$$

$$\frac{1}{2} \left(-\frac{\sqrt{3}}{2}\right) - \frac{1}{2} \left(\frac{1}{2}\right)$$

$$\frac{-1 - \sqrt{3}}{2\sqrt{2}}$$

$$7) \cos\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$$

$$\cos \frac{5\pi}{3} \cos \frac{\pi}{4} - \sin \frac{5\pi}{3} \sin \frac{\pi}{4}$$

$$\left(\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{1 + \sqrt{3}}{2\sqrt{2}}$$

$$8) \cos\left(\frac{7\pi}{6} + \frac{\pi}{4}\right)$$

$$\cos \frac{7\pi}{6} \cos \frac{\pi}{4} - \sin \frac{7\pi}{6} \sin \frac{\pi}{4}$$

$$\left(-\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{-\sqrt{3} + 1}{2\sqrt{2}}$$

$$9) \sin 75$$

$$\sin(45 + 30)$$

$$\sin 45 \cos 30 + \cos 45 \sin 30$$

$$\left(\frac{1}{\sqrt{2}}\right) \left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$10) \sin 195$$

$$\sin(45 + 150)$$

$$\sin 45 \cos 150 + \cos 45 \sin 150$$

$$\left(\frac{1}{\sqrt{2}}\right) \left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right) \left(\frac{1}{2}\right)$$

$$\frac{-\sqrt{3} + 1}{2\sqrt{2}}$$

$$11) \sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$$

$$\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4}$$

$$\left(\frac{\sqrt{3}}{2}\right) \left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right) \left(\frac{1}{\sqrt{2}}\right)$$

$$\frac{\sqrt{3} + 1}{2\sqrt{2}}$$

$$12) \sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)$$

$$\sin \frac{\pi}{4} \cos \frac{4\pi}{3} + \cos \frac{\pi}{4} \sin \frac{4\pi}{3}$$

$$\left(\frac{1}{\sqrt{2}}\right) \left(-\frac{1}{2}\right) + \left(\frac{1}{\sqrt{2}}\right) \left(-\frac{\sqrt{3}}{2}\right)$$

$$\frac{-1 - \sqrt{3}}{2\sqrt{2}}$$

13) $\tan\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

$\sin\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

$\sin\frac{5\pi}{3} \cos\frac{\pi}{4} + \cos\frac{5\pi}{3} \sin\frac{\pi}{4}$

$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\frac{-\sqrt{3}+1}{2\sqrt{2}}$

$\cos\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

$\cos\frac{5\pi}{3} \cos\frac{\pi}{4} - \sin\frac{5\pi}{3} \sin\frac{\pi}{4}$

$\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\frac{1+\sqrt{3}}{2\sqrt{2}}$

$\frac{-\sqrt{3}+1}{1+\sqrt{3}}$

14) $\tan 105^\circ$

$\tan(45+60)$

$\sin(45+60)$

$\sin 45 \cos 60 + \cos 45 \sin 60$

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2} + \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$\cos 45 \cos 60 + \sin 45 \sin 60$

$\frac{1}{\sqrt{2}} \cdot \frac{1}{2} - \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2}$

$\frac{1+\sqrt{3}}{1-\sqrt{3}}$

15) $\tan\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

$\sin\frac{5\pi}{3} \cos\frac{\pi}{4} + \cos\frac{5\pi}{3} \sin\frac{\pi}{4}$

$\cos\frac{5\pi}{3} \cos\frac{\pi}{4} + \sin\frac{5\pi}{3} \sin\frac{\pi}{4}$

$\frac{\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}$

$\frac{-\sqrt{3}+1}{1+\sqrt{3}}$

16) $\tan\left(\frac{4\pi}{3} + \frac{3\pi}{4}\right)$

$\sin\frac{4\pi}{3} \cos\frac{3\pi}{4} + \cos\frac{4\pi}{3} \sin\frac{3\pi}{4}$

$\cos\frac{4\pi}{3} \cos\frac{3\pi}{4} - \sin\frac{4\pi}{3} \sin\frac{3\pi}{4}$

$\frac{\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) + \left(-\frac{1}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}{\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{2}}\right) - \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)}$

$\frac{\sqrt{3}-1}{1+\sqrt{3}}$

17) $\cos 2(35^\circ) = 1 - 2\sin^2 35^\circ$

TRUE

18) $\cos 2(36^\circ) = 2\cos^2(60^\circ) - 1$

FALSE

19) $\sin(-40^\circ) = 2\sin(-20^\circ)\cos(-20^\circ)$

TRUE

20) $\sin 2(36^\circ) = 2\sin 72^\circ \cos 72^\circ$

FALSE

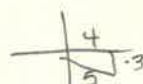
21) $\tan 2(35^\circ) = \frac{2\tan 70^\circ}{1-\tan^2 35^\circ}$

FALSE

22) $\tan(-70^\circ) = \frac{2\tan(-35^\circ)}{1-\tan^2(35^\circ)}$

TRUE

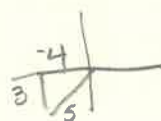
23) $\cos a = \frac{4}{5}$ Q II, find $\sin 2a$

 $\sin 2a = 2\sin a \cos a$

$= 2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$= -\frac{24}{25}$

24) $\sin \theta = \frac{3}{5}$, Q III, find $\cos 2\theta$



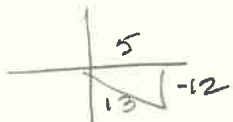
$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25}$

$= \frac{7}{25}$

25) $\cos u = \frac{5}{13}$, Q IV find $\tan 2u$



$$\begin{aligned}\frac{\sin 2u}{\cos 2u} &= \frac{2 \sin u \cos u}{\cos^2 u - \sin^2 u} \\ &= \frac{2 \left(\frac{5}{13}\right) \left(-\frac{12}{13}\right)}{\left(\frac{5}{13}\right)^2 - \left(-\frac{12}{13}\right)^2} \\ &= \frac{-120}{169} \\ &= \frac{-119}{169} \\ &= \frac{120}{119}\end{aligned}$$

26) $\cos \theta = \frac{4}{5}$ Q I find $\cos \frac{\theta}{2}$



$$\begin{aligned}\cos \frac{\theta}{2} &= + \sqrt{\frac{1 + \cos \theta}{2}} \\ &= \sqrt{\frac{1 + \frac{4}{5}}{2}} \\ &= \sqrt{\frac{9}{10}} \\ &= \frac{3}{\sqrt{10}}\end{aligned}$$

27) $\tan a = 2$, Q II find $\tan \left(\frac{a}{2}\right)$



$$\begin{aligned}\tan \frac{a}{2} &= \sqrt{\frac{1 - \cos a}{1 + \cos a}} \\ &= + \sqrt{\frac{1 + \frac{1}{\sqrt{5}}}{1 + \frac{1}{\sqrt{5}}}} \\ &= \sqrt{\frac{\sqrt{5} + 1}{\sqrt{5} - 1}}\end{aligned}$$

$$\frac{1 - \cos a}{\sin a}$$

$$\frac{1 + \frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}}$$

$$\frac{\sqrt{5} + 1}{2}$$

$$\frac{\sin a}{1 + \cos a}$$

$$\frac{\frac{2}{\sqrt{5}}}{1 + \frac{1}{\sqrt{5}}}$$

$$\frac{2}{\sqrt{5} + 1}$$

28) $\cos u = \frac{1}{\sqrt{2}}$ Q I

find $\sin \frac{u}{2}$

$$\begin{aligned}\sin \frac{u}{2} &= + \sqrt{\frac{1 - \cos u}{2}} \\ &= \sqrt{\frac{1 - \frac{1}{\sqrt{2}}}{2}} \\ &= \sqrt{\frac{\sqrt{2} - 1}{2}}\end{aligned}$$