EXERCISES 3.2

In Exercises 1–60, verify the given identity.

1. $\sin t \csc t = 1$
2. $\tan y \cot y = 1$
3. $(1 + \sin \alpha)(1 - \sin \alpha) = \cos^2 \alpha$
4. $\cot^2 y(\sec^2 y - 1) = 1$
5. $\cos^2 \beta - \sin^2 \beta = 1 - 2 \sin^2 \beta$
6. $\cos^2 \beta - \sin^2 \beta = 2 \cos^2 \beta - 1$
7. $\tan^2 \theta + 4 = \sec^2 \theta + 3$
8. $2 - \sec^2 z = 1 - \tan^2 z$
9. $\sin^2 \alpha - \sin^4 \alpha = \cos^2 \alpha - \cos^4 \alpha$
10. $\cos x + \sin x \tan x = \sec x$
11. $\dfrac{\sec^2 x}{\tan x} = \sec x \csc x$
12. $\dfrac{\cot^3 t}{\csc t} = \cos t(\csc^2 t - 1)$
13. $\dfrac{\cot^2 t}{\csc t} = \csc t - \sin t$
14. $\dfrac{1}{\sin x} - \sin x = \dfrac{\cos^2 x}{\sin x}$
15. $\sin^{1/2} x \cos x - \sin^{5/2} x \cos x = \cos^3 x \sqrt{\sin x}$
16. $\sec^6 x(\sec x \tan x) - \sec^4 x(\sec x \tan x) = \sec^5 x \tan^3 x$
17. $\dfrac{1}{\sec x \tan x} = \csc x - \sin x$
18. $\dfrac{\sec \theta - 1}{1 - \cos \theta} = \sec \theta$
19. $\cos x + \sin x \tan x = \sec x$
20. $\sec x - \cos x = \sin x \tan x$
21. $\csc x - \sin x = \cos x \cot x$
22. $\dfrac{\sec x + \tan x}{\sec x - \tan x} = (\sec x + \tan x)^2$
23. $\dfrac{1}{\tan x} + \dfrac{1}{\cot x} = \tan x + \cot x$
24. $\dfrac{1}{\sin x} - \dfrac{1}{\csc x} = \csc x - \sin x$
25. $\dfrac{\cos \theta \cot \theta}{1 - \sin \theta} - 1 = \csc \theta$
26. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$
27. $\dfrac{1}{\cot x + 1} + \dfrac{1}{\tan x + 1} = 1$
28. $\cos x - \dfrac{\cos x}{1 - \tan x} = \dfrac{\sin x \cos x}{\sin x - \cos x}$
29. $2 \sec^2 x - 2 \sec^2 x \sin^2 x - \sin^2 x - \cos^2 x = 1$
30. $\csc x(\csc x - \sin x) + \dfrac{\sin x - \cos x}{\sin x} + \cot x = \csc^2 x$
31. $2 + \cos^2 x - 3 \cos^4 x = \sin^2 x(2 + 3 \cos^2 x)$
32. $4 \tan^4 x + \tan^2 x - 3 = \sec^2 x(4 \tan^2 x - 3)$
33. $\csc^4 x - 2 \csc^2 x + 1 = \cot^4 x$
34. $\sin x(1 - 2 \cos^2 x + \cos^4 x) = \sin^5 x$
35. $\sec^4 \theta - \tan^4 \theta = 1 + 2 \tan^2 \theta$
36. $\csc^4 \theta - \cot^4 \theta = 2 \csc^2 \theta - 1$
37. $\dfrac{\sin \beta}{1 - \cos \beta} = \dfrac{1 + \cos \beta}{\sin \beta}$
38. $\dfrac{\cot \alpha}{\csc \alpha - 1} = \dfrac{\csc \alpha + 1}{\cot \alpha}$
39. $\dfrac{\tan^3 \alpha - 1}{\tan \alpha - 1} = \tan^2 \alpha + \tan \alpha + 1$
40. $\dfrac{\sin^3 \beta + \cos^3 \beta}{\sin \beta + \cos \beta} = 1 - \sin \beta \cos \beta$
41. $\cos\left(\dfrac{\pi}{2} - x\right)\csc x = 1$
42. $\dfrac{\cos[(\pi/2) - x]}{\sin[(\pi/2) - x]} = \tan x$
43. $\dfrac{\csc(-x)}{\sec(-x)} = -\cot x$
44. $(1 + \sin y)[1 + \sin(-y)] = \cos^2 y$
45. $\dfrac{\cos(-\theta)}{1 + \sin(-\theta)} = \sec \theta + \tan \theta$
46. $\dfrac{1 + \sec(-\theta)}{\sin(-\theta) + \tan(-\theta)} = -\csc \theta$
47. $\dfrac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y} = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}$
48. $\dfrac{\tan x + \tan y}{1 - \tan x \tan y} = \dfrac{\cot x + \cot y}{\cot x \cot y - 1}$
49. $\dfrac{\tan x + \cot y}{\tan x \cot y} = \tan y + \cot x$
50. $\dfrac{\cos x - \cos y}{\sin x + \sin y} + \dfrac{\sin x - \sin y}{\cos x + \cos y} = 0$
51. $\sqrt{\dfrac{1 + \sin \theta}{1 - \sin \theta}} = \dfrac{1 + \sin \theta}{|\cos \theta|}$
52. $\sqrt{\dfrac{1 - \cos \theta}{1 + \cos \theta}} = \dfrac{1 - \cos \theta}{|\sin \theta|}$
53. $\ln|\tan \theta| = \ln|\sin \theta| - \ln|\cos \theta|$
54. $\ln|\sec \theta| = -\ln|\cos \theta|$
55. $-\ln(1 + \cos \theta) = \ln(1 - \cos \theta) - 2 \ln|\sin \theta|$
56. $-\ln|\sec \theta + \tan \theta| = \ln|\sec \theta - \tan \theta|$

Ex 3.2 WS

Verify

① $\sin x \csc x = 1$

$\sin x \left(\frac{1}{\sin x}\right)$

1

② $\tan y \cot y = 1$

$\tan y \frac{1}{\tan y} =$

1

③ $(1-\sin\alpha)(1+\sin\alpha) = \cos^2\alpha$

$1 - \sin^2\alpha =$

$\cos^2\alpha$

④ $\cot^2 y(\sec^2 y - 1) = 1$

$\frac{\cos^2 y}{\sin^2 y}\left(\frac{1}{\cos^2 y} - 1\right)$

$\frac{1}{\sin^2 y} - \frac{\cos^2 y}{\sin^2 y}$

$\frac{\sin^2 y}{\sin^2 y}$

1

⑤ $\cos^2\beta - \sin^2\beta = 1 - 2\sin^2\beta$

$1 - \sin^2\beta - \sin^2\beta$

$1 - 2\sin^2\beta$

⑥ $\cos^2\beta - \sin^2\beta = 2\cos^2\beta - 1$

$\cos^2\beta - (1 - \cos^2\beta)$

$\cos^2\beta - 1 + \cos^2\beta$

$2\cos^2\beta - 1$

⑦ $\tan^2\theta + 4 = \sec^2\theta + 3$

$\sec^2 - 1 + 4$

$\sec^2\theta + 3$

⑧ $2 - \sec^2 x = 1 - \tan^2 x$

$2 - (\tan^2 x + 1)$

$-\tan^2 x + 1$

⑨ $\sin^2\theta - \sin^4\theta = \cos^2\theta - \cos^4\theta$

$\sin^2\theta(1 - \sin^2\theta)$

$1 - \cos^2\theta(\cos^2\theta)$

$\cos^2\theta - \cos^4\theta$

⑩ $\cos x + \sin x \tan x = \sec x$

$\cos x + \frac{\sin^2 x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\cos x}$

$\frac{1}{\cos x}$

$\sec x$

⑪ $\frac{\sec^2 x}{\tan x} = \sec x \csc x$

$\frac{1 + \tan^2 x}{\tan x}$

$\frac{1}{\tan x} + \tan x$

$\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$\frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$\sec x \csc x$

13 $\frac{\cot^2 x}{\csc x} = \csc x - \sin x$

$\frac{\csc^2 x - 1}{\csc x}$

$\csc x - \sin x$

(16) $\sec^4x(\sec x+\tan x)-\sec^4x(\sec x+\tan x)=\sec^3x\tan^3x$

$\sec^4x(\sec x+\tan x)(\sec^2x-1)$

$\sec^5x+\tan x \qquad \tan^2x$

$\sec^5x\tan^3x$

(17) $\dfrac{1}{\sec x+\tan x}=\csc x-\sin x$

$\dfrac{1}{\frac{1}{\cos x}+\frac{\sin x}{\cos x}}$

$\dfrac{\cos^3x}{\sin x}$

$\dfrac{1-\sin^2x}{\sin x}$

$\dfrac{1}{\sin x}-\sin x$

$\csc x-\sin x$

(20) $\sec x-\cos x=\sin x\tan x$

$\dfrac{1}{\cos x}-\cos x$

$\dfrac{1-\cos^2x}{\cos x}$

$\dfrac{\sin^2x}{\cos x}$

$\sin x\tan x$

(21) $\csc x-\sin x=\cos x\cot x$

$\dfrac{1}{\sin x}-\sin x$

$\dfrac{1-\sin^2x}{\sin x}$

$\dfrac{\cos^2x}{\sin x}$

$\cos x\cot x$

(25) $\dfrac{\cos\theta\cot\theta}{1-\sin\theta}-1=\csc\theta$

$\dfrac{\frac{\cos^2\theta}{\sin\theta}}{1-\sin\theta}$

$\dfrac{\frac{1-\sin^2\theta}{\sin\theta}}{1-\sin\theta}$

$\dfrac{(1-\sin\theta)(1+\sin\theta)}{\sin\theta}-1\dfrac{1}{\tan\theta\cos\theta}$

$\dfrac{1+\sin\theta}{\sin\theta}-\dfrac{\sin\theta}{\sin\theta}$

$\dfrac{1}{\sin\theta}$

$\csc\theta$

(26) $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=2\sec\theta$

$\dfrac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{(\cos\theta)(1+\sin\theta)}$

$\dfrac{2+2\sin\theta}{(\cos\theta)(1+\sin\theta)}$

$\dfrac{2}{\cos\theta}$

$2\sec\theta$

(29) $2\sec^2x-2\sec^2x\sin^2x-\sin^2x-\cos^2x=$

$2\sec^2x(1-\sin^2x)-(\sin^2x+\cos^2x)$

$2\sec^2x(\cos^2x)-1$

$2-1$

1

(43) $\dfrac{\csc(-x)}{\sec(-x)}=-\cot x$

$\dfrac{-\sin x}{}=-\cot x$