GO COUGARS!

p 467 Homework Questions

In Exercises 1–8, find the dot product of **u** and **v**.

5. $\mathbf{u} = 4\mathbf{i} - 2\mathbf{j}$
$\mathbf{v} = \mathbf{i} - \mathbf{j}$

7. $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j}$
$\mathbf{v} = -2\mathbf{i} - 3\mathbf{j}$

In Exercises 9–18, use the vectors $\mathbf{u} = \langle 2, 2 \rangle$, $\mathbf{v} = \langle -3, 4 \rangle$, and $\mathbf{w} = \langle 1, -2 \rangle$ to find the indicated quantity. State whether the result is a vector or a scalar.

9. $\mathbf{u} \cdot \mathbf{u}$

11. $(\mathbf{u} \cdot \mathbf{v})\mathbf{v}$

13. $(3\mathbf{w} \cdot \mathbf{v})\mathbf{u}$

17. $(\mathbf{u} \cdot \mathbf{v}) - (\mathbf{u} \cdot \mathbf{w})$

In Exercises 25–34, find the angle θ between the vectors.

25. $\mathbf{u} = \langle 1, 0 \rangle$
$\mathbf{v} = \langle 0, -2 \rangle$

27. $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}$
$\mathbf{v} = -2\mathbf{j}$

31. $\mathbf{u} = 5\mathbf{i} + 5\mathbf{j}$
$\mathbf{v} = -6\mathbf{i} + 6\mathbf{j}$

33. $\mathbf{u} = \cos\left(\frac{\pi}{3}\right)\mathbf{i} + \sin\left(\frac{\pi}{3}\right)\mathbf{j}$
$\mathbf{v} = \cos\left(\frac{3\pi}{4}\right)\mathbf{i} + \sin\left(\frac{3\pi}{4}\right)\mathbf{j}$

In Exercises 39–42, use vectors to find the interior angles of the triangle with the given vertices.

39. $(1, 2), (3, 4), (2, 5)$

In Exercises 43–46, find $\mathbf{u} \cdot \mathbf{v}$, where θ is the angle between **u** and **v**.

43. $\|\mathbf{u}\| = 4, \|\mathbf{v}\| = 10, \theta = \frac{2\pi}{3}$

45. $\|\mathbf{u}\| = 9, \|\mathbf{v}\| = 36, \theta = \frac{3\pi}{4}$

In Exercises 47–52, determine whether **u** and **v** are orthogonal, parallel, or neither.

47. $\mathbf{u} = \langle -12, 30 \rangle$
$\mathbf{v} = \langle \frac{1}{2}, -\frac{5}{4} \rangle$

49. $\mathbf{u} = \frac{1}{4}(3\mathbf{i} - \mathbf{j})$
$\mathbf{v} = 5\mathbf{i} + 6\mathbf{j}$

51. $\mathbf{u} = 2\mathbf{i} - 2\mathbf{j}$
$\mathbf{v} = -\mathbf{i} - \mathbf{j}$

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GO COUGARS!

p 478 Homework Questions

In Exercises 1–6, plot the complex number and find its absolute value.

1. $-7i$ **2.** -7
3. $-4 + 4i$ **4.** $5 - 12i$
5. $6 - 7i$ **6.** $-8 + 3i$

In Exercises 11–30, represent the complex number graphically, and find the trigonometric form of the number.

11. $3 - 3i$ **12.** $2 + 2i$
13. $\sqrt{3} + i$ **14.** $4 - 4\sqrt{3}i$
15. $-2(1 + \sqrt{3}i)$ **16.** $\frac{5}{2}(\sqrt{3} - i)$
17. $-5i$ **18.** $4i$
19. $-7 + 4i$ **20.** $3 - i$

In Exercises 31–40, represent the complex number graphically, and find the standard form of the number.

31. $3(\cos 120° + i \sin 120°)$

37. $8\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

38. $7(\cos 0 + i \sin 0)$

39. $3[\cos(18° 45') + i \sin(18° 45')]$

In Exercises 47–58, perform the operation and leave the result in trigonometric form.

47. $\left[2\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)\right]\left[6\left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)\right]$

48. $\left[\frac{3}{4}\left(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}\right)\right]\left[4\left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}\right)\right]$

49. $\left[\frac{5}{3}(\cos 140° + i \sin 140°)\right]\left[\frac{2}{3}(\cos 60° + i \sin 60°)\right]$

53. $\frac{\cos 50° + i \sin 50°}{\cos 20° + i \sin 20°}$

57. $\frac{12(\cos 52° + i \sin 52°)}{3(\cos 110° + i \sin 110°)}$

In Exercises 59–66, (a) write the trigonometric forms of the complex numbers, (b) perform the indicated operation using the trigonometric forms, and (c) perform the indicated operation using the standard forms, and check your result with that of part (b).

59. $(2 + 2i)(1 - i)$ **60.** $(\sqrt{3} + i)(1 + i)$
61. $-2i(1 + i)$ **62.** $4(1 - \sqrt{3}i)$
63. $\frac{3 + 4i}{1 - \sqrt{3}i}$ **64.** $\frac{1 + \sqrt{3}i}{6 - 3i}$
65. $\frac{5}{2 + 3i}$

[redacted]

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p 479 # Homework Questions

In Exercises 71–88, use DeMoivre's Theorem to find the indicated power of the complex number. Write the result in standard form.

71. $(1 + i)^5$

77. $[5(\cos 20° + i \sin 20°)]^3$

79. $\left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right)^{12}$

83. $(3 - 2i)^5$

87. $\left[2\left(\cos \frac{\pi}{10} + i \sin \frac{\pi}{10}\right)\right]^5$

88. $\left[2\left(\cos \frac{\pi}{8} + i \sin \frac{\pi}{8}\right)\right]^6$

In Exercises 89–104, (a) use the theorem on page 476 to find the indicated roots of the complex number, (b) represent each of the roots graphically, and (c) write each of the roots in standard form.

89. Square roots of $5(\cos 120° + i \sin 120°)$

90. Square roots of $16(\cos 60° + i \sin 60°)$

91. Cube roots of $8\left(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3}\right)$

92. Fifth roots of $32\left(\cos \frac{5\pi}{6} + i \sin \frac{5\pi}{6}\right)$

93. Square roots of $-25i$

94. Fourth roots of $625i$

95. Cube roots of $-\frac{125}{2}(1 + \sqrt{3}i)$

96. Cube roots of $-4\sqrt{2}(1 - i)$

97. Fourth roots of 16

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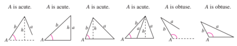
Chapter 6 Review Topics

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Ambiguous Case

The Ambiguous Case (SSA)

Consider a triangle in which you are given a, b, and A. $(h = b \sin A)$



	A is acute.	*A* is acute.	*A* is acute.	*A* is acute.	*A* is obtuse.	*A* is obtuse.
Sketch						
Necessary condition	$a < h$	$a = h$	$a > b$	$h < a < b$	$a \le b$	$a > b$
Triangles possible	None	One	One	Two	None	One

Area

Area of an Oblique Triangle

The area of any triangle is one-half the product of the lengths of two sides times the sine of their included angle. That is,

$$\text{Area} = \frac{1}{2}bc \sin A = \frac{1}{2}ab \sin C = \frac{1}{2}ac \sin B.$$

Heron's Area Formula

Given any triangle with sides of lengths a, b, and c, the area of the triangle is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = (a + b + c)/2$.

Law of Cosines

Law of Cosines

Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc \cos A$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac \cos B$	$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab \cos C$	$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Vectors

Component Form of a Vector

The component form of the vector with initial point $P = (p_1, p_2)$ and terminal point $Q = (q_1, q_2)$ is given by

$$\overrightarrow{PQ} = \langle q_1 - p_1, q_2 - p_2 \rangle = \langle v_1, v_2 \rangle = \mathbf{v}.$$

The **magnitude** (or length) of $\mathbf{v}$ is given by

$$\|\mathbf{v}\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2} = \sqrt{v_1^2 + v_2^2}.$$

If $\|\mathbf{v}\| = 1$, $\mathbf{v}$ is a **unit vector.** Moreover, $\|\mathbf{v}\| = 0$ if and only if $\mathbf{v}$ is the zero vector $\mathbf{0}$.

$$\mathbf{u} = \text{unit vector} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \left(\frac{1}{\|\mathbf{v}\|}\right)\mathbf{v}.$$

The unit vectors $\langle 1, 0 \rangle$ and $\langle 0, 1 \rangle$ are called the **standard unit vectors** and are denoted by

$$\mathbf{i} = \langle 1, 0 \rangle \quad \text{and} \quad \mathbf{j} = \langle 0, 1 \rangle$$

$$\mathbf{u} = \langle x, y \rangle = \langle \cos \theta, \sin \theta \rangle = (\cos \theta)\mathbf{i} + (\sin \theta)\mathbf{j}$$

Definition of the Dot Product

The **dot product** of $\mathbf{u} = \langle u_1, u_2 \rangle$ and $\mathbf{v} = \langle v_1, v_2 \rangle$ is

$$\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2.$$

Angle Between Two Vectors

If θ is the angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$, then

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{u}\| \|\mathbf{v}\|}.$$

complex numbers

Definition of the Absolute Value of a Complex Number

The **absolute value** of the complex number $z = a + bi$ is

$$|a + bi| = \sqrt{a^2 + b^2}.$$

Trigonometric Form of a Complex Number

The **trigonometric form** of the complex number $z = a + bi$ is

$$z = r(\cos \theta + i \sin \theta)$$

where $a = r \cos \theta$, $b = r \sin \theta$, $r = \sqrt{a^2 + b^2}$, and $\tan \theta = b/a$. The number r is the **modulus** of z, and θ is called an **argument** of z.

Product and Quotient of Two Complex Numbers

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ be complex numbers.

$$z_1z_2 = r_1r_2[\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)] \quad \text{Product}$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)], \quad z_2 \ne 0 \quad \text{Quotient}$$

DeMoivre's Theorem

If $z = r(\cos \theta + i \sin \theta)$ is a complex number and n is a positive integer, then

$$z^n = [r(\cos \theta + i \sin \theta)]^n = r^n(\cos n\theta + i \sin n\theta).$$

Finding *n*th Roots of a Complex Number

For a positive integer n, the complex number $z = r(\cos \theta + i \sin \theta)$ has exactly n distinct nth roots given by

$$\sqrt[n]{r}\left(\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n}\right)$$

where $k = 0, 1, 2, \ldots, n - 1$.

Apr 10-2:13 PM

Chapter 6 Review

Task Cards

1. Pick a partner
2. Pick a problem
3. Work it together

Repeat steps 2. and 3.

Apr 10-2:15 PM

HOMEWORK

p 482

9-14, 17-22, 25-34, 37-67 odd, 71, 72,

73-87 odd, 97-111 odd

p 486 1-17, 20-24

Workbook p 86-87

Aug 29-6:38 AM

Trig Review Word Problems Worksheet

1. h = 1053.15
2. Denver to Omaha 468 miles

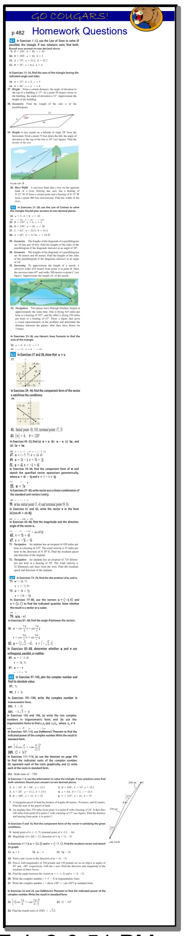
   bearing 252.23
3. 37.77 hours, bearing 104.41
4. 740.5 knots per hour, bearing 32.14
5. PR = 502.14 cm, P = 28.12, R = 19.38
6. smallest angle = 27.66, K = 16.25
7. bearing = 76.15
8. speed = 574.7 mph, bearing = 337.9
9. A = 86.29, b = 5.35, c = 3.71, K = 9.9
10. C = 36.3, A = 49.8, T = 93.9, K = 9.3
11. I = 137.47 / 4.53

    G = 23.53 / 156.47

    i = 10.62 / 1.24

    K = 10.9 or 1.27
12. e = 43.25, H = 110.06, S = 13.26, K = 251.5

Apr 17-3:11 PM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p 482 Homework Questions

Feb 2-9:51 PM

GO COUGARS!

pg 486 **Review Questions**

In Exercises 1–6, use the information to solve the triangle. If two solutions exist, find both solutions. Round your answers to two decimal places.

1. $A = 24°,\ B = 68°,\ a = 12.2$
2. $B = 104°,\ C = 33°,\ a = 18.1$
3. $A = 24°,\ a = 11.2,\ b = 13.4$
4. $a = 4.0,\ b = 7.3,\ c = 12.4$
5. $B = 100°,\ a = 15,\ b = 23$
6. $C = 123°,\ a = 41,\ b = 57$

7. A triangular parcel of land has borders of lengths 60 meters, 70 meters, and 82 meters. Find the area of the parcel of land.
8. An airplane flies 370 miles from point A to point B with a bearing of 24°. It then flies 240 miles from point B to point C with a bearing of 37° (see figure). Find the distance and bearing from point A to point C.

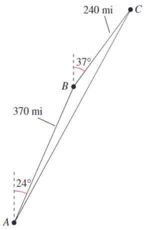

FIGURE FOR 8

In Exercises 9 and 10, find the component form of the vector v satisfying the given conditions.

9. Initial point of **v**: $(-3, 7)$; terminal point of **v**: $(11, -16)$
10. Magnitude of **v**: $\|\mathbf{v}\| = 12$; direction of **v**: $\mathbf{u} = \langle 3, -5\rangle$

In Exercises 11–13, $\mathbf{u} = \langle 3, 5\rangle$ and $\mathbf{v} = \langle -7, 1\rangle$. Find the resultant vector and sketch its graph.

11. $\mathbf{u} + \mathbf{v}$
12. $\mathbf{u} - \mathbf{v}$
13. $5\mathbf{u} - 3\mathbf{v}$

14. Find a unit vector in the direction of $\mathbf{u} = \langle 4, -3\rangle$.
15. Forces with magnitudes of 250 pounds and 130 pounds act on an object at angles of 45° and −60°, respectively, with the x-axis. Find the direction and magnitude of the resultant of these forces.

Feb 2-9:51 PM

Review Practice Problems

1. Use the given vectors for the following: $v = \langle -2, 3\rangle \quad w = \langle 5, 1\rangle$
   a. sketch w - v
   b. find the unit vector for vector v
   c. find the trig component form of vector w (calc ok)

Apr 15-6:00 AM

2. How many triangles with given information can be formed?

Do not solve.

a. $A = 61^\circ$, $a = 8$, $b = 21$

b. $A = 112^\circ$, $a = 15$, $b = 17$

c. $B = 18^\circ$, $C = 65^\circ$, $c = 12$

Apr 7-6:38 AM

3. Solve the triangle to two decimal places.

$a = 7$, $b = 15$, $c = 19$

4. Twelve horses are equally spaced on a merry-go-round. If the chord connecting the center of each horse is 18 feet long, what is the diameter of the merry-go-round? What is the length of the arc between each horse?

Apr 8-6:08 AM

5. Solve the triangle.

   B = 35, b = 12 , c = 15

6. Solve the triangles given the following information.

   a. C = 75 , b = 49, c = 48

Apr 13-1:38 PM