

### 7.6 The Determinant and Inverse of a 2 x 2 Matrix

To solve $\frac{a x}{a}=\frac{b}{a}$
We can multiply by $a^{-1}=\frac{1}{a}$

$$
\frac{1}{a} a x=b\left(\frac{1}{a}\right)
$$

## To solve a matrix we use the same process

$A=\left[\begin{array}{cc}3 & -3 \\ -2 & 2\end{array}\right]$

Ex find AB and $\mathrm{BA} \quad A=\left[\begin{array}{cc}2 & -1 \\ -3 & 1\end{array}\right] \quad B=\left[\begin{array}{ll}-1 & -1 \\ -3 & -2\end{array}\right]$
$A B=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$ identity matrix $\Rightarrow A \& B$ are inverses of each other
$B A=\left[\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right]$

$$
A B=B A
$$

because $A \& B$ are inverses.

Finding an inverse matrix:
Ex2 Find the inverse of $A=\left[\begin{array}{cc}1 & -2 \\ -1 & 3\end{array}\right]$
We know that $[A]\left[\mathrm{A}^{-1}\right]=[I]$

$$
\begin{aligned}
& {\left[\begin{array}{cc}
1 & -2 \\
-1 & 3
\end{array}\right]\left[\begin{array}{ll}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{array}\right]=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]} \\
& {\left[\begin{array}{cc}
1 & -2 \\
-1 & 3
\end{array}\right]\left[\begin{array}{ll}
a & b \\
c & d
\end{array}\right]=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]} \\
& a-2 c=1 \quad b-2 d=0 \\
& -a+3 c=0 \quad-b+3 d=1 \\
& c=1 \quad d=1 \\
& a=3 \quad b=2 \\
& A^{-1}=\left[\begin{array}{ll}
3 & 2 \\
1 & 1
\end{array}\right] \quad A A^{-1}=\left[\begin{array}{cc}
1 & -2 \\
-1 & 3
\end{array}\right]\left[\begin{array}{ll}
3 & 2 \\
1 & 1
\end{array}\right]=\left[\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right]
\end{aligned}
$$

There is an easier way to find an inverse matrix! (Thank goodness!)

But first we need to talk about the determinant of a matrix.
If $A=\left[\begin{array}{ll}a & b \\ c & d\end{array}\right]$ then $\operatorname{det}[\mathrm{A}]=a d-b c$
Ex3 Find the determinant of $A=\left[\begin{array}{cc}1 & -2 \\ -1 & 3\end{array}\right]$

$$
|A|=1
$$

Now, to find an inverse matrix use:

$$
\frac{1}{a d-b c}\left[\begin{array}{cc}
d & -b \\
-c & a
\end{array}\right]
$$

Ex Find the inverse of the matrix

$$
A=\left[\begin{array}{cc}
1 & -2 \\
-1 & 3
\end{array}\right] \begin{aligned}
\text { 1. find }|A| & =1 \\
\text { 2. find } A^{-1} & =\frac{1}{1}\left[\begin{array}{ll}
3 & 2 \\
1 & 1
\end{array}\right] \\
A^{-1} & =\left[\begin{array}{ll}
3 & 2 \\
1 & 1
\end{array}\right]
\end{aligned}
$$

Ex5 Find the inverse of $A=\left[\begin{array}{ll}3 & 6 \\ 0 & 2\end{array}\right]$ Check your answer.

$$
\begin{aligned}
|A| & =3 \cdot 2-6 \cdot 0 \\
& =6 \\
A^{-1} & =\frac{1}{6}\left[\begin{array}{cc}
2 & -6 \\
0 & 3
\end{array}\right] \\
& =\left[\begin{array}{cc}
\frac{1}{3} & -1 \\
0 & \frac{1}{2}
\end{array}\right]
\end{aligned}
$$

## What if the determinant is zero?

A singular matrix is one whose determinant is zero. If the determinant is zero the matrix does not have an inverse or is invertible.


Find the determinant
Ex6

$$
A=\left[\begin{array}{ll}
3 & 6 \\
1 & 2
\end{array}\right]
$$

Ex7
$A=\left[\begin{array}{cc}2 & -4 \\ -2 & 4\end{array}\right]$

What do you notice?
7.6 Inverses and Determinants of Square Matrix.notebook

p 547 3, 5, 11-15 odd, 29, 33-39 odd,

$$
49,53
$$

Worksheet 1-10

$$
\begin{array}{cc}
A X=B & X A=B \\
A^{-1} A X=A^{-1} B & X A R^{-1}=B A^{-1}
\end{array}
$$

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