Review - Chapter 9 Highlights

Polar Points

graph

rename

$r < 0$

$\theta < 0$

$2\pi < \theta < 4\pi$

rectangular → polar

points (r, θ)

- find r (magnitude)
- find θ using $\tan\theta = y/x$

equations

- $r^2 = x^2 + y^2$
- $x = r\cos\theta$
- $y = r\sin\theta$

polar → rectangular

points (x, y)

- $(r\cos\theta, r\sin\theta)$

equations

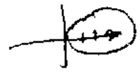
- get ~~to~~ r^2
  - by squaring
  - by multiplying by r
- given θ, take tan of both sides

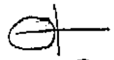
$\downarrow \rightarrow = \frac{y}{x}$

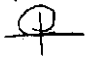
cross multiply

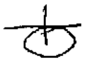
Graphs of polar

$r = 3$ - circle $C(0,0)$

$r = 6\cos\theta$ 

$6\sin\theta$ 

$-6\cos\theta$ 

$-6\sin\theta$ 

circle w/ radius of 3

$r = 2 \pm 2\sin\theta$

$2 \pm 2\cos\theta$

cardioid

$r = 3 \pm 4\sin\theta$

$3 \pm 4\cos\theta$

loopy limacon

$r = 4 \pm 3\sin\theta$

$4 \pm 3\cos\theta$

dimpled limacon

$r = 5\sin 3\theta$

rose

$\theta = \frac{\pi}{3}$ line

$r\sin\theta = 2$ horizontal line

$r\cos\theta = 2$ vertical line

Chapter 9 Review

Find the rectangular equation by eliminating the parameter. Sketch the graph and state the domain and range of the rectangular equation.

1. $x = 3t + 2$
   $y = t + 1$

2. $x = t^2 + 4$
   $y = t^2 - 4$

3. $x = \sqrt{2t}$
   $y = 4t$

4. $x = 2\cos t$
   $y = 3 - 2\sin t$

5. $x = 1 - 2\cos t$
   $y = 2 + 5\sin t$

May 7-5:42 AM

6. Plot the polar point and state 3 equivalent points with the following constraints.

$\left(5, \frac{5\pi}{3}\right)$

$r < 0,\ 0 \le \theta < 2\pi$

$r > 0,\ -2\pi \le \theta < 0$

$r > 0,\ 2\pi \le \theta < 4\pi$

7. Convert from rectangular coordinates to polar coordinates.

a) $(6, 150°)$

b) $\left(-3, -\frac{3\pi}{4}\right)$

c) $(-3.2, 185°)$ calc ok

May 7-5:51 AM

8. Find the polar coordinates for the following rectangular points.

a) $\left(-1, \sqrt{3}\right)$

b) $(4, 0)$

c) $(-5, -7)$ calc ok

May 7-6:02 AM

9. Convert the rectangular equation to a polar equation.

a) $2x^2+2y^2=5$ | b) $2xy=1$ | c) $x=12$

May 7-6:10 AM

10. Convert the polar equation to a polar equation.

a) $r=-3\sin\theta$ | b) $r=2$ | c) $\theta=\frac{4\pi}{3}$ | d) $r=\sin\theta-\cos\theta$

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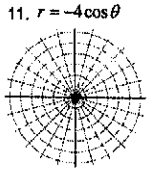
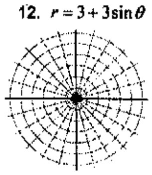
Graph the polar equations.

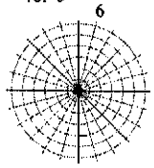
11. $r=-4\cos\theta$



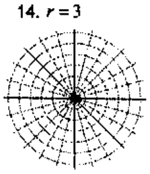
12. $r=3+3\sin\theta$



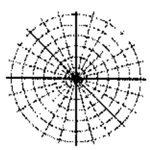
13. $\theta=\frac{11\pi}{6}$



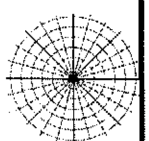
14. $r=3$



15. $r=5-2\sin\theta$



16. $r=1+3\cos\theta$



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Additional Questions

1. $x = t + 2$, $y = \sqrt{t}$
2. $x = \sqrt{t} + 4$, $y = \sqrt{t} - 4$
3. $x = 2\sin y$, $y = 1 + 2\cos t$
4. $x = 5 + 3\cos t$, $y = 2 + \sin t$

5. Plot and state 3 equivalent points for $\left(-3, \frac{7\pi}{6}\right)$.
6. Convert to rectangular: #6 without calc and $(2, 2.5)$ with calc.
7. Convert to polar: $(-\sqrt{3}, 1)$, $(1, -1)$.
8. Convert to polar: $xy = 2$
   $2x - y = 3$
   $y = -4$
9. Convert to rectangular: $\theta = \frac{4\pi}{3}$
   $r = 2$
   $r = -2\sin\theta$

May 7-6:26 AM

Ch 9 Review Key

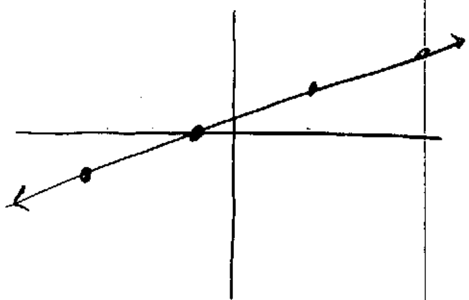
1) $x = 3t+2$ $\quad D(-\infty,\infty)$ $\quad D_t(-\infty,\infty)$
$y = t+1$ $\quad D(-\infty,\infty)$

t	x	y
-2	-4	-1
-1	-1	0
0	2	1
1	5	2
2	8	3

$D(-\infty,\infty)$ $\quad R(-\infty,\infty)$

$\frac{x-2}{3} = t \qquad y = \frac{x-2}{3}+1$

$= \frac{1}{3}x + \frac{1}{3}$



2) $x = t^2+4$ $\quad D_x(-\infty,\infty)$ $\quad D_t(-\infty,\infty)$
$y = t^2-4$ $\quad D_y(-\infty,\infty)$

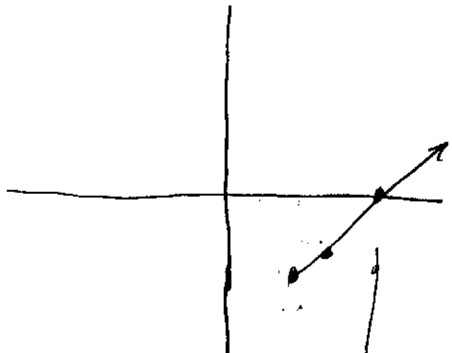
t	x	y
-2	8	0
-1	5	-3
0	4	-4
1	5	-3
2	8	0

$D[4,\infty)$ $\quad R[-4,\infty)$

$x-4 = t^2$

$y = x-4-4$

$y = x-8$



3) $x = \sqrt{2t}$ $\quad D_x[0,\infty)$ $\quad D_t[0,\infty)$
$y = 4t$ $\quad D_y[-\infty,\infty)$

t	x	y
0	0	0
2	2	8
8	4	32

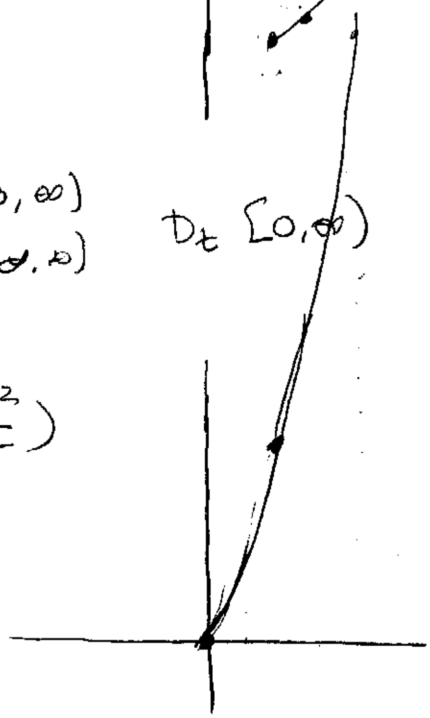
$D[0,\infty)$ $\quad R[0,\infty)$

$x^2 = 2t$

$\frac{x^2}{2} = t$

$y = 4\left(\frac{t^2}{2}\right)$

$= 2t^2$



4) $x = 2\cos t$

$y = 3 - 2\sin t$

same coeff $\Rightarrow$ circle!

t	x	y
$-\frac{\pi}{2}$	0	5
0	2	3
$\frac{\pi}{2}$	0	1
π	-2	3

$D\ [-2, 2]$

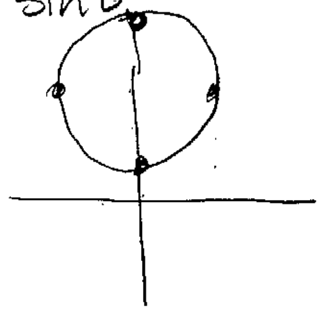
$R\ [1, 5]$

$\frac{x}{2} = \cos t \qquad \frac{y-3}{-2} = \sin t$

$\frac{x^2}{4} = \cos^2 t \qquad \frac{(y-3)^2}{4} = \sin^2 t$

$\frac{x^2}{4} + \frac{(y-3)^2}{4} = 1$

$x^2 + (y-3)^2 = 4$



5) $x = 1 - 2\cos t$

$y = 2 + 5\sin t$

diff coeff $\Rightarrow$ ellipse!

t	x	y
$-\frac{\pi}{2}$	1	-3
0	-1	2
$\frac{\pi}{2}$	1	7
π	3	2

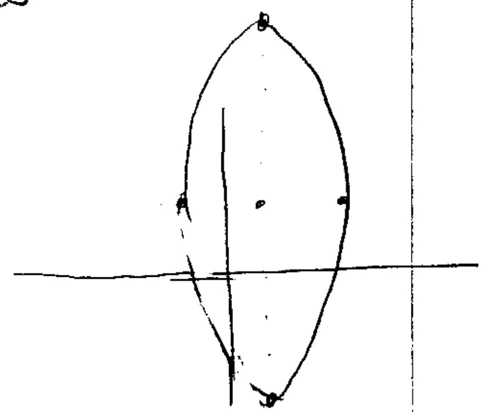
$D\ [-1, 3]$

$R\ [-3, 7]$

$\frac{x-1}{-2} = \cos t \qquad \frac{y-2}{5} = \sin t$

$\frac{(x-1)^2}{4} = \cos^2 t \qquad \frac{(y-2)^2}{25} = \sin^2 t$

$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{25} = 1$



6) $\left(-5, \frac{2\pi}{3}\right)$ $\left(5, -\frac{\pi}{3}\right)$ $\left(5, \frac{11\pi}{3}\right)$

7) a) $(-3\sqrt{3}, 3)$ b) $\left(\frac{13}{\sqrt{2}}, -\frac{13}{\sqrt{2}}\right)$ c) $(3.19, .28)$

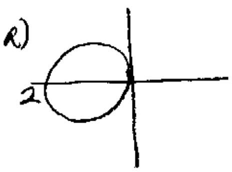
8) a) $\left(2, \frac{2\pi}{3}\right)$ b) $(4, 0)$ c) $(\sqrt{74}, 234.46)$

9 a) $r = \sqrt{5/2}$ b) $r^2 = \frac{1}{2}\sec\theta\csc\theta$ c) $r = 12\sec\theta$

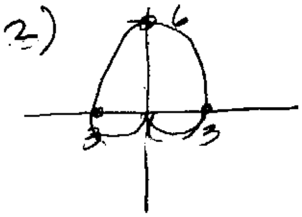
10 a) $x^2 + (y + 2/3)^2 = \frac{9}{4}$ b) $x^2 + y^2 = 4$

c) $y = \sqrt{3}\,x$ d) $(x + \frac{1}{2})^2 + (y - \frac{1}{2})^2 = \frac{1}{2}$

11) a)



12)



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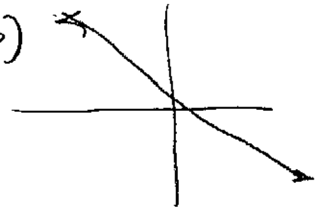

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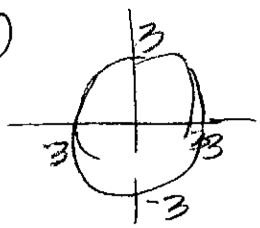

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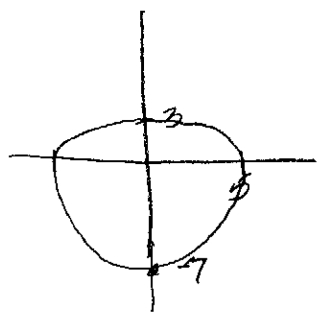

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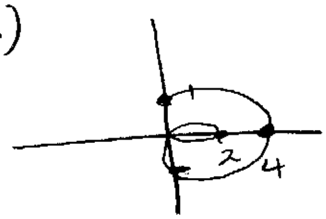
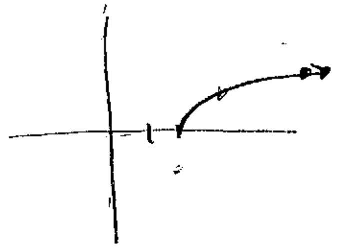


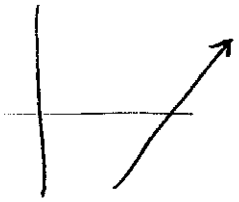
Additional Questions

1) 

D $[-2, \infty)$
R $[0, \infty)$

$y = \sqrt{x-2}$

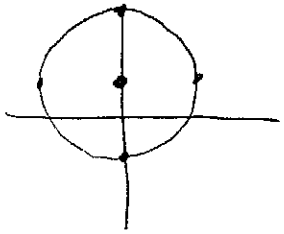
t	x	y
0	2	0
1	3	1
4	6	2

2) 

D $[4, \infty)$
R $[-4, \infty)$

$y = x - 8$

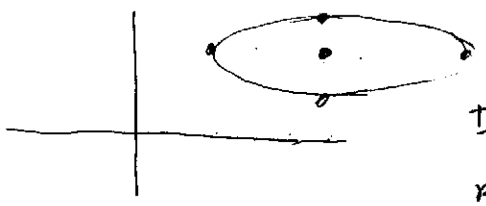
t	x	y
0	4	-4
1	5	-3
4	6	-2
9	7	-1

3) 

D $[-2, 2]$
R $[-1, 3]$

$x^2 + (y-1)^2 = 4$

t	x	y
$-\frac{\pi}{2}$	-2	1
0	0	3
$\frac{\pi}{2}$	2	1
π	0	-1

4) 

D $[2, 8]$
R $[1, 3]$

$\frac{(x-5)^2}{9} + \frac{(y-2)^2}{1} = 1$

t	x	y
$-\frac{\pi}{2}$	5	1
0	8	2
$\frac{\pi}{2}$	5	3
π	2	2

5) $\left(-3, \frac{19\pi}{6}\right)$ $\left(-3, -\frac{5\pi}{6}\right)$ $\left(3, \frac{\pi}{6}\right)$

6) $\left(-\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$ $(-1.6, 1.2)$

7) $\left(2, \frac{5\pi}{6}\right)$ $\left(\sqrt{7}, \frac{7\pi}{4}\right)$

8) $r^2 = 2\sec\theta\csc\theta$

$r = \frac{3}{2\cos\theta - \sin\theta}$

$r = -4\csc\theta$

9) $y = \sqrt{3}x$

$x^2 + y^2 = 4$

$x^2 + (y+1)^2 = 1$