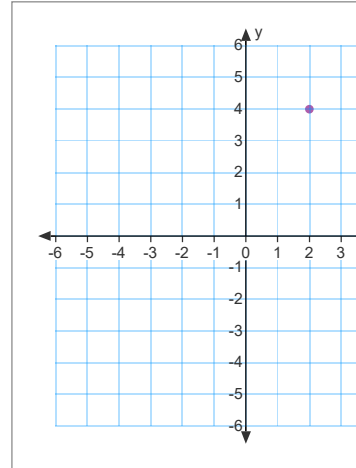


10.3 The Ellipse.notebook

Warm up

- Find the equation of a parabola with latus rectum length of 8 ending at (2, -1) and (2, 7) and opens to the left.
- Find the equation of the parabola with a directrix $y = 2$ and focus (2, 4).
- Find the focus point of $x = \frac{1}{16}(y + 7)^2 - 3$



Dec 4-7:13 AM

GO COUGARS!

p 740 Homework Questions

In Exercises 1-16, match the equation with its graph. The graphs are labeled (a) through (l).

(a) $x^2 = 16$

(b) $x^2 = -16$

(c) $x^2 = (y-2)^2 - 16$

(d) $x^2 = (y+2)^2 - 16$

(e) $x^2 = (y-2)^2 + 16$

(f) $x^2 = (y+2)^2 + 16$

(g) $x^2 = (y-2)^2 - 4$

(h) $x^2 = (y+2)^2 - 4$

(i) $x^2 = (y-2)^2 + 4$

(j) $x^2 = (y+2)^2 + 4$

(k) $x^2 = (y-2)^2 - 8$

(l) $x^2 = (y+2)^2 - 8$

(m) $x^2 = 16$

(n) $x^2 = -16$

(o) $x^2 = (y-2)^2 - 16$

(p) $x^2 = (y+2)^2 - 16$

(q) $x^2 = (y-2)^2 + 16$

(r) $x^2 = (y+2)^2 + 16$

(s) $x^2 = (y-2)^2 - 4$

(t) $x^2 = (y+2)^2 - 4$

(u) $x^2 = (y-2)^2 + 4$

(v) $x^2 = (y+2)^2 + 4$

(w) $x^2 = (y-2)^2 - 8$

(x) $x^2 = (y+2)^2 - 8$

In Exercises 17-24, find the vertex, focus, and directrix of the parabola and sketch its graph.

17. $x^2 = 16y$ 18. $x^2 = -16y$
 19. $x^2 = 4y$ 20. $x^2 = -4y$
 21. $x^2 = (y-2)^2 - 16$ 22. $x^2 = (y+2)^2 - 16$
 23. $x^2 = (y-2)^2 + 16$ 24. $x^2 = (y+2)^2 + 16$

In Exercises 25-28, find the vertex, focus, and directrix of the parabola. Use a graphing utility to graph the parabola.

25. $x^2 + 4x + 4 = y$
 26. $x^2 - 6x + 9 = y$
 27. $x^2 + 4x + 4 = y$
 28. $x^2 - 6x + 9 = y$

In Exercises 29-34, find the standard form of the equation of the parabola with the given characteristics and vertex at the origin.

29. Focus: (1, 0)
 30. Focus: (0, -2)
 31. Directrix: $x = 1$

In Exercises 35-41, find the standard form of the equation of the parabola with the given characteristics.

35. Vertex: (2, 2); focus: (3, 2)
 36. Vertex: (-1, 2); focus: (-1, 4)
 37. Vertex: (0, 2); focus: (0, 4)
 38. Vertex: (1, 2); focus: (1, 4)
 39. Vertex: (2, 2); focus: (2, 4)
 40. Vertex: (3, 2); focus: (3, 4)
 41. Vertex: (4, 2); focus: (4, 4)

In Exercises 42-44, find the standard form of the equation of the parabola for the given point and the focus or directrix.

42. $(-2, 0)$; focus: (2, 0)
 43. $(-2, 0)$; directrix: $x = 2$
 44. $(-2, 0)$; focus: (2, 0)

45. Area Project Find an area under a parabola. Consider the parabola $y = ax^2$ for $a > 0$. The area under the parabola from $x = -1$ to $x = 1$ is $\frac{2}{3}a$. The area under the parabola from $x = -1$ to $x = 1$ is $\frac{2}{3}a$. The area under the parabola from $x = -1$ to $x = 1$ is $\frac{2}{3}a$.

46. Area Project Find an area under a parabola. Consider the parabola $y = ax^2$ for $a > 0$. The area under the parabola from $x = -1$ to $x = 1$ is $\frac{2}{3}a$. The area under the parabola from $x = -1$ to $x = 1$ is $\frac{2}{3}a$. The area under the parabola from $x = -1$ to $x = 1$ is $\frac{2}{3}a$.

Handwritten notes:

$y - \frac{5}{4} = \frac{1}{4}(x^2 - 2x + 2)$
 $y - \frac{5}{4} = \frac{1}{4}(x-1)^2$
 $y = \frac{1}{4}(x-1)^2 + \frac{5}{4}$
 $p = 1$

$y^2 - 6y + 9 = -8x - 15 + 9$
 $(y-3)^2 = -8x - 6$
 $(y-3)^2 + 6 = -8x$
 $\frac{1}{8}(y-3)^2 - 2 = x$

$x^2 + 4x + 4 = -6y + 2 = -6$
 $(x+2)^2 = -6y + 2$
 $(x+2)^2 = -6y + 6$
 $\frac{1}{6}(x+2)^2 = 1 - y$

$y = \frac{1}{16}x^2$
 $\frac{1}{16}x^2 = \frac{1}{4}(x-2)^2$
 $x^2 = 4(x-2)^2$
 $x^2 = 4(x^2 - 4x + 4)$
 $x^2 = 4x^2 - 16x + 16$
 $-3x^2 + 16x - 16 = 0$
 $3x^2 - 16x + 16 = 0$
 $x = \frac{16 \pm \sqrt{256 - 192}}{6} = \frac{16 \pm \sqrt{64}}{6} = \frac{16 \pm 8}{6}$
 $x = \frac{24}{6} = 4$ or $x = \frac{8}{6} = \frac{4}{3}$
 $y = \frac{1}{16}(4)^2 = \frac{1}{4}$ or $y = \frac{1}{16}(\frac{4}{3})^2 = \frac{1}{9}$

$p = \frac{a^2}{4a} = \frac{a}{4}$
 $a = \frac{1}{16}$
 $p = \frac{1/16}{4} = \frac{1}{64}$
 $y = \frac{1}{64}x^2$

Feb 2-9:51 PM

10.3 The Ellipse

Find the equation given points

Find the points given an equation

Nov 9-8:15 AM

Definition of Ellipse

An **ellipse** is the set of all points (x, y) in a plane, the sum of whose distances from two distinct fixed points (**foci**) is constant. [see below](#)

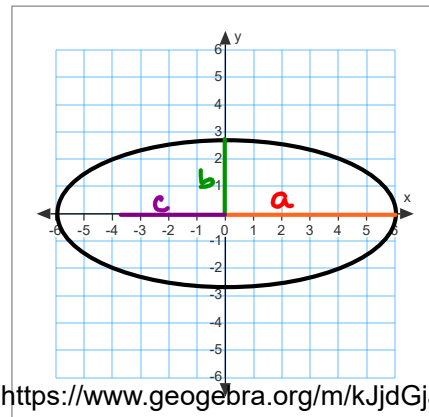
middle to vertex = a

major axis = $2a$ vertices are on major axis

middle to side = b

minor axis = $2b$

middle to focus = c foci on major axis



<https://www.geogebra.org/m/kJjdGjJQ>

Nov 8-12:27 PM

10.3 The Ellipse.notebook

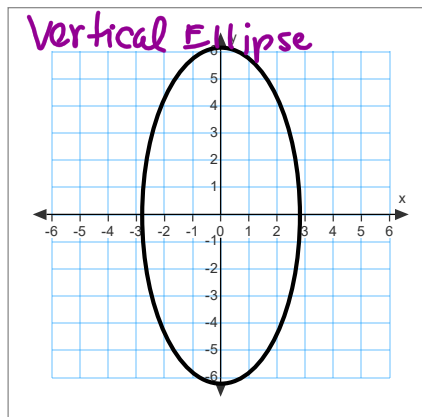
$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$a > b > 0$$

$$a^2 - b^2 = c^2$$

horizontal ellipse

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$



Nov 30-2:27 PM

Find equation of an ellipse with

$$F_1(-2, 2) \quad F_2(4, 2)$$

Major axis length of 10

$$a = 5$$

$$m = (1, 2) \quad c = 3$$

$$F = (-2, 2)$$

$$a^2 - b^2 = c^2$$

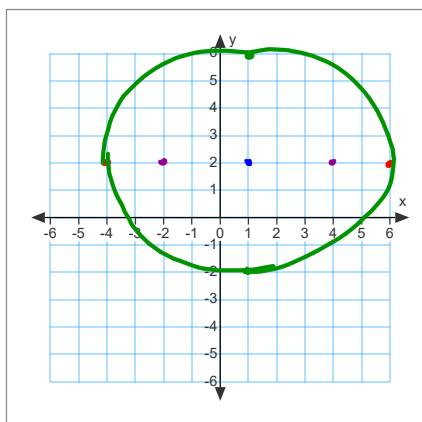
$$\frac{(x-1)^2}{25} + \frac{(y-2)^2}{16} = 1$$

$$5^2 - b^2 = 3^2$$

$$-b^2 = 9 - 25$$

$$b^2 = 16$$

$$b = 4$$



Nov 8-12:36 PM

10.3 The Ellipse.notebook

Find the middle, foci and vertices for

$9x^2 + 4y^2 + 36x - 8y + 4 = 0$ and sketch.

$$9x^2 + 36x + 4y^2 - 8y = -4$$

$$9(x^2 + 4x + 2^2) + 4(y^2 - 2y + 1^2) = -4 + 9(4) + 4(1)$$

$$\frac{9(x+2)^2}{36} + \frac{4(y-1)^2}{36} = \frac{36}{36}$$

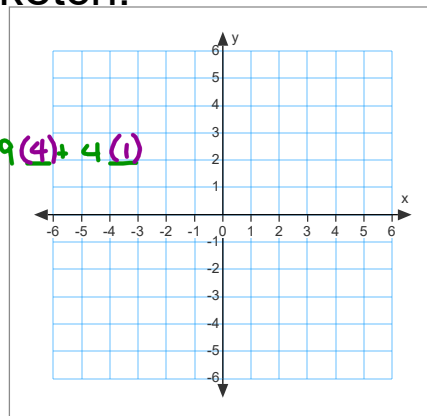
$$\frac{(x+2)^2}{4} + \frac{(y-1)^2}{9} = 1 \quad a=3$$

$$b=2$$

$$m(-2, 1) \quad c=\sqrt{5}$$

$$F(-2, 1 \pm \sqrt{5})$$

$$V(-2, 4) (-2, -2)$$



Nov 30-3:16 PM

Now you try! Find the middle, vertices and foci

$$16x^2 + 25y^2 - 32x - 50y + 16 = 0$$

$$16(x^2 - 2x + 1) + 25(y^2 - 2y + 1) = -16 + 16 + 25$$

$$\frac{16(x-1)^2}{25} + \frac{25(y-1)^2}{25} = \frac{25}{25}$$

$$\frac{(x-1)^2}{25/16} + \frac{(y-1)^2}{1} = 1 \quad a=5/4$$

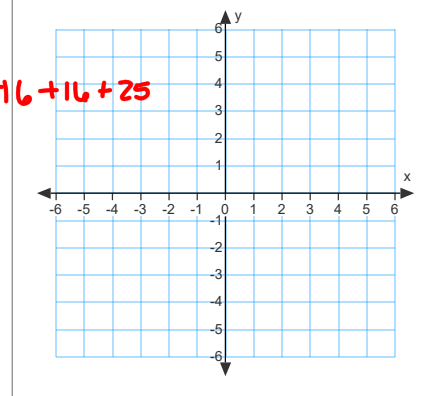
$$b=1$$

$$m(1, 1)$$

$$F\left(\frac{7}{4}, 1\right) \left(\frac{1}{4}, 1\right)$$

$$c=3/4$$

$$V\left(\frac{9}{4}, 1\right) \left(-\frac{1}{4}, 1\right)$$



Nov 30-3:18 PM

10.3 The Ellipse.notebook

Planetary orbits

apogee - the greatest distance from an object orbiting around another

perigee - the smallest distance from an object orbiting around another

the object that is traversed around sits at a focus.

Apr 14-8:21 AM

The first artificial satellite to orbit the earth was Sputnik (launched by Russia in 1957). Its orbit was elliptical with the center of the earth at one focus. The major and minor axes of the orbits had lengths of 13,906 km and 13,887 km, respectively. Find the apogee and perigee from Earth's center to the satellite. Use these to find the least distance and greatest distance of the satellite from Earth's surface in this orbit. (Earth has a radius of 6378 km.)



$$\begin{aligned} 2a &= 13906 & 2b &= 13887 & c^2 &= 6953^2 - 6943.5^2 \\ a &= 6953 & b &= 6943.5 & c &= 363.34 \end{aligned}$$

$$\begin{aligned} \text{apogee} &= a + c \\ &= 7316 \text{ km} \end{aligned}$$

$$\begin{aligned} \text{perigee} &= a - c \\ &= 6590 \text{ km} \end{aligned}$$

nearest to earth perigee - earth's radius
938 km

farthest from earth apogee - earth's radius
212 km

Apr 14-8:24 AM

10.3 The Ellipse.notebook

A circle is a special type of ellipse where $a = b$!

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \quad \text{or} \quad x^2 + y^2 = a^2$$

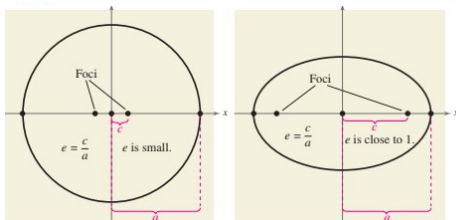
Eccentricity

One of the reasons it was difficult for early astronomers to detect that the orbits of the planets are ellipses is that the foci of the planetary orbits are relatively close to their centers, and so the orbits are nearly circular. To measure the ovalness of an ellipse, you can use the concept of **eccentricity**.

Definition of Eccentricity

The **eccentricity** e of an ellipse is given by the ratio

$$e = \frac{c}{a}$$



The closer the eccentricity is to 0 the more circular the ellipse is.

What is the eccentricity of the previous orbit example?

Nov 8-12:46 PM

HOMework



p 750 1-6, 13-19 odd, 25, 29,

41, 45, 47, 51, 55-59 odd

Feb 2-9:51 PM

10.3 The Ellipse.notebook

Discuss $\frac{x^2}{25} + \frac{y^2}{4} = 1$

Discuss changes in equation for vertical ellipse

Nov 8-12:40 PM

Nov 4-10:10 AM